NAVPERS 93400A-8

FUNDAMENTALS OF ELECTRONICS

VOLUME 8

TABLES AND MASTER INDEX

Bureau of Naval Personnel

PREFACE

This book is part of a nine-volume set entitled "Fundamentals of Electronics". The nine-volumes include:

Volume 1a - NavPers 93400A-1a, Basic Electricity, Direct Current
Volume 1b - NavPers 93400A-1b, Basic Electricity, Alternating Current
Volume 2 - NavPers 93400A-2, Power Supplies and Amplifiers
Volume 3 - NavPers 93400A-3, Transmitter Circuit Applications
Volume 4 - NavPers 93400A-4, Receiver Circuit Applications
Volume 5 - NavPers 93400A-5, Oscilloscope Circuit Applications
Volume 6 - NavPers 93400A-6, Microwave Circuit Applications
Volume 7 - NavPers 93400A-7, Electromagnetic Circuits and Devices
Volume 8 - NavPers 93400A-8, Tables and Master Index

If you are becoming acquainted with electricity or electronics for the first time, study volumes one through seven in their numerical sequence. If you have background equivalent to the information contained in volumes one and two, you are prepared to study the material contained in any of the remaining volumes. A master index for all volumes is included in volume eight. Volume eight also contains technical and mathematical tables that are useful in the study of the other volumes.

A question (or questions) follows each group of paragraphs. The questions are designed to determine if you understand the immediately preceding information. As you study, write out your answers to each question on a sheet of paper. If you have difficulty in phrasing an answer, restudy the applicable paragraphs. Do not advance to the next block of paragraphs until you are satisfied that you have written a correct answer.

When you have completed study of the text matter and written satisfactory answers to all questions on two facing pages of the book, compare your answers with those at the top of the next even-numbered page. If the answers match, you may continue your study with reasonable assurance that you have understood and can apply the material you have studied. Whenever your answers are incorrect, restudy the applicable material to determine why the book answer is correct and yours is not. If you make an honest effort to follow these instructions, you will have achieved the maximum learning benefits from each study assignment.

Follow the directions of your instructor in answering the review questions included at the end of each chapter.

TABLE OF CONTENTS

CHAPTER 59

MATHEMATICAL OPERATIONS

It took the human race centuries to develope mathematical concepts which are considered simple by today's standards. Developing a number system to accommodate the ever-growing needs of man was a long, hard struggle. There were some interesting milestones on this road of progress.

One of these milestones was the idea of number POSITION and PLACE VALUE. In the number, 333, the three at the right means three things or three units. The middle three means three times ten or 30, and the three to the left means 3 times one hundred or 300. The discovery of place value gave civilization a terrific spurt, for with this system, it was possible to write any number however large or small.

The idea of ZERO on our number scale was another milestone on the road to progress. In fact, zero has been called one of man's greatest inventions. This statement is not as fantastic as it sounds. If it were not for zero, numbers such as 100 would be difficult to indicate.

59-1. Addition

When adding WHOLE NUMBERS such as 1, 2, 3, etc., they must be written according to our place value system - units to units, tens to tens, etc. The tens are added to tens, hundreds to hundreds, etc.

To ADD, begin at the top of the units column and add down. The units column may be found in the example problem.

EXAMPLE:

```
              ┌──────thousands
              │  ┌───hundreds
              │  │ ┌─tens
          5,749◄──units
            123
              5
         14,657
         20,534
```

It is often helpful to mentally add the numbers as one is progressing down through the column. The sum of the units column is 24. The number, 24, is composed of 4 units and 2 tens. Therefore, a 4 placed in the units column below the line, and a 2 is "CARRIED OVER" to the tens column where it will be added to the numbers in the tens column. The tens column is then added. The sum of the tens column (including the 2 which was "carried over") is 13. The 3 is written in the tens column, and the 1 is "carried over" to the hundreds column. This process is continued till all of the numbers in each column are added.

EXERCISE 1:

Add the numbers in the following problems:

1.	631	2.	68	3.	462	4.	4,379
	222		723		321		323
	31		11		8,921		182

5. 7,221,692	6. 3,256	7. 889	8. 218
341,222	2,445	3,654	666

9. 60,000	10. 756
3,500	234
	785

59-2. Subtraction

SUBTRACTION is the operation of finding the difference between two numbers. This is the same as finding the amount that must be added to one number, called the SUBTRAHEND, to equal another number, called the MINUEND.

In subtraction, as in addition, the units must be placed under the units, the tens under the tens, etc.

EXAMPLE:

Subtract 684 from 992.

992 is equal to 9 hundreds, 9 tens, and 2 units

684 is equal to 6 hundreds, 8 tens, and 4 units

Since four units cannot be subtracted from two units, one tens value is "BORROWED" from the tens column and added to the units column.

Therefore:

992 is equal to	9 hundreds,	8 tens,	12 units
684 is equal to	6 hundreds,	8 tens,	4 units
308	3 hundreds,	0 tens,	8 units

Hence, 684 subtracted from 992 is equal to 308.

EXERCISE 2:

Subtract the following:

1. 42	2. 683	3. 6,011	4. 564
33	672	2,133	223

5. 49	6. 786	7. 831	8. 322
26	427	165	231

9. 888	10. 781
349	392

59-3. Multiplication

MULTIPLICATION is defined as the operation of adding a number to itself a given number of times. Therefore, 4 X 8 (four times eight) could be thought of as adding 8 four times.

The number that is to be multiplied is called the MULTIPLICAND, and the number of times it is

to be added is called the MULTIPLIER. The answer obtained from performing the operation of multiplication is called the PRODUCT.

EXAMPLE:

6 times 3 means 6 + 6 + 6 equals 18

$$\begin{array}{r} 6 \text{ multiplicand} \\ \times\ 3 \text{ multiplier} \\ \hline 18 \text{ product} \end{array}$$

To multiply the number 683 by 4, place the multiplier under the multiplicand so that the units will be under the units, the tens under the tens, etc.

$$\begin{array}{r} 1 \\ \hline 683 \\ 4 \\ \hline 2 \end{array}$$

The operation of multiplication is performed as follows: Begin at the right by multiplying the unit digit of the multiplicand by the unit digit of the multiplier. Three units times four units is equal to 12 units which is equivalent to 1 tens and 2 units. Therefore, write the 2 in the units column of the product and "CARRY" the one to the tens column of the multiplicand where it will be added to the tens column of the product after the tens column of the multiplicand is multiplied by the multiplier.

Next, multiply the 4 times the 8 and add the carried one, 4 x 8 = 32. Adding the one 32 + 1 = 33 which is 3 hundreds and three tens. Place the three tens in the tens column of the product and carry the 3 hundred to the hundreds column of the multiplicand.

$$\begin{array}{r} 31 \\ \hline 683 \\ 4 \\ \hline 2,732 \end{array}$$

Repeating the same process, multiply the 4 times the 6 and the carried 3 is added to that product (4 x 6) + 3 = 27. This figure, 27, is equal to 2 thousands, and 7 hundreds. Place the 7 in the hundreds column of the product, and carry the 2. Since this operation completes the multiplication, the 2 should be placed in the thousands column of the product. Therefore, the product of 683 times 4 is equal to 2,732.

EXERCISE 3:

Perform the following operations:

1.	36 x5	2.	40 x4	3.	95 x2	4.	87 x3
5.	536 x4	6.	3,467 x5	7.	48 x86	8.	453 x864
9.	6,584 x28	10.	805 x306				

59-4. Division

DIVISION is the operation of determining how many times a given number, called a DIVIDEND, contains another number called a DIVISOR. Division may also be stated as an operation of repeated

subtractions. The resultant, or answer, is called a QUOTIENT. The signs which indicate division are:

$$\div \qquad\qquad - \qquad\qquad / \qquad\qquad \overline{\smash{\big)}}$$

Therefore, 6 divided by 3 may be written as $6 \div 3$, $\frac{6}{3}$, $6/3$, or $3\overline{\smash{\big)}6}$.

A simple procedure for dividing is as follows:

Step One:

To find the quotient of 47.9 divided by 7.24, place the dividend under the line of the division sign. Place the divisor before the division sign.

$$7.24\overline{\smash{\big)}47.9}$$

Step Two:

Move the decimal point in the divisor to the right of the divisor's extreme right digit counting the number of places moved. Move the decimal point of the dividend to the right the same number of places. Use zeros to fill the places at which the dividend has no value. Place a decimal point above the line over the decimal point which was moved in the dividend.

$$7\varkappa24.\overline{\smash{\big)}47\varkappa90.}$$

Step Three:

Determine the number of times the divisor will go into the dividend. Start by seeing if the divisor will go into the digit (4). Since it will not, try the digits (47). Keep this up until the divisor will go at least one time. 724 will go into 4790 approximately 6 times. Place the 6 over the line above the last digit of the dividend that was used in this step. Multiply the 6 by the divisor and subtract the product from the digits of the dividend used in this step. This difference should be less than the divisor. If not, increase the number in the quotient by one.

$$
\begin{array}{r}
6. \\
724\overline{\smash{\big)}4790.} \\
\underline{4344} \\
446
\end{array}
$$

Step Four:

The next digit in the dividend is understood to be zero. Therefore, this zero is brought down and placed next to the difference obtained in step three. Determine how many times 724 can be divided into 4460. Place this number, 6, in the quotient to the right of the decimal point. Multiply the multiplier by this number and subtract the product from 4460 obtaining the new difference of 116.

$$
\begin{array}{r}
6.6 \\
724\overline{\smash{\big)}4790.0} \\
\underline{4344} \\
4460 \\
\underline{4344} \\
116
\end{array}
$$

Step Five:

Repeat step four two more times thereby carrying the division out three places which will be considered sufficiently accurate for most work in electricity. If the value of the final remainder is equal to half of the divisor or more, increase the last digit by one. This is called "ROUNDING UP". If

the difference is less than half do not change the quotient (round down). If extreme accuracy is required, indicate that the final difference is the remainder 116. Always check the final answer by multiplying the quotient by the divisor and adding the final difference of 116.

EXERCISE 4:

Performing the following operations: (carry out to two significant figures)

1. $645 \div 9$

2. $\dfrac{249}{8}$

3. $\dfrac{289}{4}$

4. $\dfrac{100}{37}$

5. $\dfrac{435}{9}$

6. $\dfrac{200}{73}$

7. $\dfrac{31,573}{1,760}$

8. $\dfrac{250}{38}$

9. $\dfrac{150}{71}$

10. $135 \div 2$

59-5. Arithmetic Mean

Many times, it is required to apply a combination of operations to find the ARITHMETIC MEANS of voltages, currents and powers. The arithmetic mean is a simple average of a group of numbers. In fact, it is often called the AVERAGE. Suppose you wish to determine the average value of your blitz grades. The sum of all of the blitz grades divided by the total number of individual grades considered will render the mean blitz grade.

EXAMPLE:

Find the arithmetic mean of the following numbers: 100, 90, 70, 80, 70, 90, and 80.

Step One:

Find the sum of the numbers.

$$
\begin{array}{r}
100 \\
90 \\
70 \\
80 \\
70 \\
90 \\
80 \\
\hline
580
\end{array}
$$

Step Two:

Divide this sum by the number of blitzes.

$$
\begin{array}{r}
82.857 \\
7\overline{)580.000} \\
56 \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50
\end{array}
$$

The average or mean of the grades is 82.857. Rounding off, the average grade is 82.86.

COMMON FRACTIONS

59-6. Definitions

A COMMON FRACTION is an indicated division, it expresses a number of equal parts into which something has been divided. As an example, 3/8 could be thought of as some object that has been divided into 3 of 8 equal parts, or as a division, 3 divided by 8.

The number under the line is called the DENOMINATOR. It shows the number of parts into which the object has been divided. The number above the line is called the NUMERATOR. It tells how many parts are taken or considered.

There are two types of fractions - PROPER and IMPROPER. A proper fraction is a fraction the numerator of which has a smaller value than the denominator. As an example, 1/2 and 3/4 are proper fractions. An improper fraction is one in which the numerator is equal to or larger than the denominator. As an example, $^4/4$, and $^9/8$ are improper fractions.

59-7. Properties of Fractions

An important principle involving fractions is that: The numerator and denominator of any fraction can be multiplied or divided by the same factor (excluding zero) without changing the value of the fraction. This is true because the number one (sometimes called UNITY) is the identity element for the operation of multiplication and division. This means that if a quantity, such as six, is multiplied or divided by one, it will not change the value of the quantity. Six times one is equal to six. Six divided by one is equal to six. Therefore, if the fraction 2/3 is multiplied by the fraction 4/4 (which is equal to one), the value of the fraction will be unchanged.

EXAMPLE:

$$2/3 \times 4/4 = 8/12$$

$$12/16 = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

59-8. Changing a Mixed or Whole Number to an Improper Fraction

By definition, a WHOLE number is a number which contains no fractions, and a MIXED NUMBER is one which contains a whole number plus a fraction. The numbers 2, 4 and 6 are whole numbers. The numbers $2\frac{1}{2}$ and $3\frac{1}{4}$ are mixed numbers.

When performing computations with whole or mixed numbers, it is sometimes necessary to change them into improper fractions. The following examples will illustrate the process involved.

EXAMPLE:

Change the number 5 to a fraction which has the number 6 in the denominator.

$$\text{since } 6/6 = 1$$

$$\text{then } 5 \times \frac{6}{6} = \frac{30}{6}$$

Change the mixed number $3\frac{1}{2}$ to a fraction which has a numerical denominator of 2.

$$\text{since } 3\frac{1}{2} = 3 + \frac{1}{2}$$

$$\text{and } \frac{2}{2} = 1$$

$$\text{then } 3\frac{1}{2} = 3 \times \frac{2}{2} + \frac{1}{2}$$

$$= \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$$

This process may be summarized by the following statements:

To change a whole number to an improper fraction, multiply the whole number by a unit fraction (fraction equal to one) with the desired denominator.

To change a mixed number to an improper fraction, change the whole number part of an improper fraction having the desired denominator, and then add the fraction.

EXERCISE 5:

1. Convert the following to a fraction which has the number nine in the denominator:

    a) 5   b) 10   c) 15   d) 32   e) 18

2. Change the following to improper fractions.

    a) $2\frac{1}{3}$ b) $3\frac{3}{4}$ c) $6\frac{1}{7}$ d) $62\frac{1}{8}$ e) $7\frac{1}{3}$

59-9. Reducing a Fraction to its Lowest Terms

Fractions reduced to their lowest terms are easier to read, have more meaning, and are much easier to work with. For these reasons, fractions are always reduced to their lowest terms.

To reduce fractions to their lowest terms, the numerator and the denominator are searched for a common number which can be divided evenly into both numbers. As an example, in the fraction 6/8, both the numerator and the denominator may be divided by 2 resulting in:

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

The process of separating a number into two or more smaller numbers having the original number as their product is called FACTORING. The smaller numbers obtained are called FACTORS of the larger number. In the last example, 6/8, the number 2 was a common factor of both the numerator and the denominator.

EXAMPLE:

$$\frac{6}{8} = \frac{3 \times 2}{4 \times 2} = \frac{3}{4} \times \frac{2}{2} = \frac{3}{4} \times 1 = \frac{3}{4}$$

Factors common to both numerator and denominator can and should be divided out.

When reducing a fraction to its lowest term, find the common factors in both the numerator and denominator and divide them out.

EXAMPLE:

$$\frac{9}{21} = \frac{3 \times 3}{3 \times 7} = \frac{3}{7}$$

Notice that there are two three's in the numerator of the previous example, and only one in the denominator. Only one pair may be divided. The quotient of the division is one. Multiplying one by any quantity is equal to the number.

59-10. Prime Factors

As previously defined, a factor of a whole number is any whole number which will divide into the whole number evenly. Thus, 2, 5 and 7 are factors of the number 70. The number one is also a factor. However, it is not normally shown. It is assumed to be present.

A PRIME NUMBER is a number which has been factored to the point where it can only be divided by itself and the number one. The numbers 2, 5, 7 and 13 are examples of prime numbers.

Utilizing the definitions of factors and prime numbers, it follows that the prime factors of a whole number is any prime number that will divide the whole number evenly. Thus, the numbers 3, 5 and 7 are prime factors of the number 105.

Locating the prime factors of a number is the process of finding the prime numbers that will exactly divide into that number. Begin this process by dividing the number by the smallest prime number 2, and continue dividing the consecutive quotients by 2 until it will not divide into the quotient evenly. Then divide the quotient by successively higher prime numbers, and continue this process until the final quotient is the number one.

EXAMPLE:

Find the prime factors of the number 60.

$$
\begin{array}{r}
2 \ \underline{/60} \\
2 \ \underline{/30} \\
3 \ \underline{/15} \\
5 \ \underline{/5} \\
1
\end{array}
$$

Therefore, the number 60, can be expressed as a product of its prime factors.

$$60 = 2 \times 2 \times 3 \times 5$$

EXERCISE 6:

Find the prime factors of the following numbers:

1. 420   2. 780   3. 12   4. 36   5. 75

59-11. Lowest Common Multiple

A MULTIPLE of a whole number is a whole number which will divide the original whole number evenly. Thus, 36 is a multiple of the number 6, and the number 24 is a multiple of the number 8.

If a whole number can be divided evenly by two or more numbers, it is a COMMON MULTIPLE. Thus, the number 36, is a common multiple of the numbers 6 and 3. The smallest number which can be divided evenly by two or more numbers is the LOWEST COMMON MULTIPLE, and is abbreviated L.C.M. The number 15 is the L.C.M. of the numbers 3 and 5.

To find the L.C.M. of two or more numbers, factor each number into its prime factors and find the product of all the different prime factors using each different factor the greatest number of times it appears in any one number.

EXAMPLE:

Find the L.C.M. of the numbers 8, 12, and 24.

Factoring into prime factors:

$$8 = 2 \times 2 \times 2$$

$$12 = 2 \times 2 \times 3$$

$$24 = 2 \times 2 \times 2 \times 3$$

The different prime factors are the numbers 2 and 3. The number 2 appears twice as the prime factor of 12, three times as the prime factor of the number 8, and three times as the prime factor of the number 24. The number 3 appears once as a prime factor of 12 and once as a prime factor of 24. Therefore, to find the L. C. M. of the group of numbers, the number 2 must be used as a factor three times and the number 3 must be used as a factor once.

Hence: L. C. M. = 2 x 2 x 2 x 3

                                                                      = 24

Find the L. C. M. of the numbers 8, 36, and 15.
Expressing each number in its prime factors:

$$8 = 2 \times 2 \times 2$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$15 = 3 \times 5$$

Therefore, the L. C. M. is:

$$L. C. M. = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

$$= 360$$

EXERCISE 7:

Find the L. C. M. of the following:

1. 10, 15 and 30   2. 9, 24 and 36   3. 12, 18 and 32

4. 6, 105 and 8   5. 20, 30 and 60

59-12. Lowest Common Denominator

The LOWEST COMMON DENOMINATOR of two or more fractions is the lowest number that is exactly divisible by the given denominators which is the L. C. M. of the denominator. The lowest common denominator is abbreviated L. C. D.

The following steps may be applied in reducing fractions to their L. C. D.'s.

Step One:

Find the L. C. D. of the denominator. This is the same process as finding the L. C. M. of a group of numbers.

Step Two:

Using each fraction - one at a time - divide the L. C. D. by the denominator of the fraction considered, and multiply both numerator and denominator of the fraction considered by the quotient thus obtained.

EXAMPLE:

Reduce $\frac{1}{6}$, $\frac{1}{9}$ and $\frac{1}{18}$ to their L. C. D.

$$L. C. D. = L. C. M.$$

The L. C. M. of the numbers 6, 9 and 18 is:

$$L. C. M. = 18$$

Changing the fractions to the lowest common denominator, 18.

$$\frac{1}{6} = \frac{1}{6} \times \frac{3}{3} = \frac{3}{18}$$

$$\frac{1}{9} = \frac{1}{9} \times \frac{2}{2} = \frac{2}{18}$$

$$\frac{1}{18} = \frac{1}{18} \times \frac{1}{1} = \frac{1}{18}$$

Reduce $\frac{1}{12}$, $\frac{1}{18}$, $\frac{1}{36}$

$$L. C. M. = L. C. D.$$

$$L. C. D. = 36$$

$$\frac{1}{12} = \frac{1}{12} \times \frac{3}{3} = \frac{3}{36}$$

$$\frac{1}{18} = \frac{1}{18} \times \frac{2}{2} = \frac{2}{36}$$

$$\frac{1}{36} = \frac{1}{36} \times \frac{1}{1} = \frac{1}{36}$$

59-13. Addition and Subtraction of Fraction

Fractions having common denominators are added or subtracted by adding or subtracting the numerators of the fractions considered. Thus, to add two or more fractions having common denominators, add the numerators and write the sum over the common denominator.

EXAMPLE:

$$\frac{2}{8} + \frac{3}{8} = \frac{2 + 3}{8} = \frac{5}{8}$$

When subtracting two fractions, subtract the numerator of the subtrahend from the numerator of the minuend and write the difference over the common denominator.

EXAMPLE:

$$\frac{5}{8} - \frac{3}{8} = \frac{5 - 3}{8} = \frac{2}{8} = \frac{1}{4}$$

When performing the operations of addition and subtraction on fractions having unlike denominators, observe the following rules:

Change the fractions to equivalent fractions having a L. C. D.

Perform the indicated operation on the numerators of the equivalent fractions.

EXAMPLE:

$$\frac{1}{8} + \frac{1}{12} + \frac{1}{24} = \frac{3}{24} + \frac{2}{24} + \frac{1}{24} = \frac{3+2+1}{24} = \frac{6}{24} = \frac{1}{4}$$

$$\frac{1}{4} - \frac{1}{8} - \frac{1}{12} = \frac{6}{24} - \frac{3}{24} - \frac{2}{24} = \frac{6-3-2}{24} = \frac{1}{24}$$

EXERCISE 8:

Perform the indicated operations:

1. 1/2 + 1/3

2. 7/8 + 1/12 + 1/24

3. 1/4 - 1/8 - 1/12

4. 4/5 + 2/10 + 4/20

5. $2\frac{1}{3} + 3\frac{1}{4}$

6. 4/5 + 20/11 + 2/8

7. $1\frac{1}{3} + \frac{6}{8}$

8. $\frac{1}{2} - \frac{1}{4}$

9. 1/5 - 1/15 - 1/30

10. 18/32 + 5/4 - 1/5

59-14. Multiplication of Fractions

To multiply two or more fractions, multiply the numerators by the numerators and the denominators by the denominators. The product of the numerators of the factors becomes the numerator of the product. The product of the denominators of the factors becomes the denominator of the product.

EXAMPLE:

$$2/5 \times 4/5 = \frac{2 \times 4}{5 \times 5} = \frac{8}{25}$$

$$1/2 \times 3/4 \times 7/8 = \frac{1 \times 3 \times 7}{2 \times 4 \times 8} = \frac{21}{64}$$

In the previous example, the order in which the multiplication took place was unimportant. The fraction 3/4 could have been multiplied by 7/8, and that product multiplied by the remaining fraction 1/2. The product would be the same.

EXERCISE 9:

Find the product of the following:

1. 5/8 x 12

2. 5 x 4/9

3. 1/3 x 2/3

4. 1/2 x 1/3 x 2/5

5. 3/4 x 6

6. 4/3 x 1/6

7. $10\frac{1}{2} \times 3\frac{1}{3}$

8. $1\frac{5}{6} \times 13$

9. $8\frac{3}{4} \times 2/5$

10. 7/12 x 3/5

59-15. Division of Fractions

To divide one fraction by another, invert the divisor (the one being divided into the other) and multiply.

EXAMPLE:

$$1/2 \div 1/3 = 1/2 \times 3/1 = 1\frac{1}{2}$$

$$2/5 \div 3/5 = 2/5 \times 5/3 = 2/3$$

$$1\frac{1}{2} \div 2/3 = 3/2 \div 2/3 = 3/2 \times 3/2 = 9/4 = 2\frac{1}{4}$$

Notice that, just as in multiplications, the first step is to change the mixed numbers to improper fractions. The second step is then to invert the divisor. The third step is to multiply, following all of the rules from multiplication.

EXERCISE 10:

Perform the following divisions:

1. $3/8 \div 2/3$ 2. $2\frac{1}{3} \div 1\frac{1}{2}$ 3. $5/8 \div 5/16$

4. $1/3 \div 4/6$ 5. $3/10 \div 3/4$ 6. $3/4 \div 3/20$

7. $14 \div 7/10$ 8. $2/9 \div 1/2$ 9. $3/4 \div 5/8$

10. $3/20 \div 5$

59-16. Decimal Fractions

The decimal system is a convenient way to write complicated numbers and mixed fractions. Decimals are easier to add, subtract, multiply and divide than fractions. For this reason, it is important to be able to convert from a decimal to a fraction and from a fraction to a decimal.

The word decimal is derived from the Latin word decum, meaning ten. Essentially, decimals are another way of writing fractions having denominators of 10, 100, 1,000, etc. For example, the number 0.3 is a fraction written in the decimal system. It represents the fraction 3/10. The period (.) between the digit 0 and the digit 3 is called a decimal point. The location of the decimal point determines whether the denominator of the fraction it represents should be 10, 100, 1,000, etc.

EXAMPLE:

$$0.3 = 3/10$$
$$0.03 = 3/100$$
$$0.003 = 3/1000$$
$$0.0003 = 3/10,000$$

Whole numbers are written in the decimal system by placing the decimal point after the last digit in the number.

EXAMPLE:

$$3 = 3.0, \quad 60 = 60.0, \quad \text{and} \quad 800 = 800.0$$

Very complicated numbers and mixed fractions can be written very easily in the decimal system. Notice the following decimals and their meanings:

$$1.680 = \frac{1,680}{1,000} = 1\frac{680}{1000}$$

$$100.8 = \frac{1,008}{10.00} = 100\frac{8}{10}$$

$$396.71 = \frac{39,671}{100} = 396\frac{71}{100}$$

$$0.0085 = 85/10,000$$

Memorize the denominators associated with a specific location of the decimal point. Notice also that the number of zeros in the denominator always equals the number of places to the left of the last digit in the decimal. This rule is true even for the decimal version of whole numbers.

EXAMPLE:

0.52 = 52/100 (decimal point two places to the left of last
                         digit, two zeros in the denominator.)

$$167.8 = 1,678/10 = 167\frac{8}{10} = 167\frac{4}{5}$$

(decimal point one place to the left of last digit - one
zero in the denominator)

59-17. Converting Fractions to Decimals

To convert fractions to decimals, perform the following operations:

Write the numerator as a decimal by adding a decimal point after the last digit.

Divide the denominator into the numerator.

Add the zeros after the decimal in the numerator as necessary.

Place a decimal point in the answer vertically above its location in the numerator.

EXAMPLE:

$$1\frac{1}{2} = 1 + \frac{1}{2}$$

$$\frac{1}{2} = 2\overline{)1.0}^{\,0.5}$$

Therefore:

$$1\frac{1}{2} = 1 + 0.5 = 1.5$$

$$1\frac{3}{32} = 1 + \frac{3}{32}$$

$$\frac{3}{32} = 32\overline{)\begin{array}{r} .09375 \\ 3.00000 \end{array}}$$

$$\begin{array}{r} 288 \\ \hline 120 \\ 96 \\ \hline 240 \\ 224 \\ \hline 160 \\ 160 \\ \hline 0 \end{array}$$

$$1\frac{3}{32} = 1 + 0.09375$$

$$1\frac{3}{32} = 1.09375$$

It can be seen from the above examples that it is extremely important to do this type of work neatly, so that the decimal point may be located with little difficulty.

Notice that sometimes it makes no difference how many zeros are added to the number being divided. It will still not divide without a remainder. In these situations division is only accomplished to meet the requirements of the problem. Sometimes the required accuracy of an answer may be out to several decimal places. In other applications an answer accurate to one decimal place will be sufficient.

59-18. Adding and Subtracting Decimals

When adding and subtracting decimals, be sure that the decimal points are arranged vertically directly over one another. Then add or subtract the number as if they were whole numbers.

To add 60.0, 0.003, 1.6 and 32.05, the numbers are arranged in columns, and the decimal points in each row occupy the same position. This is shown in the example.

EXAMPLE:

$$\begin{array}{r} 60.0 \\ 0.003 \\ 1.6 \\ 32.05 \\ \hline 93.653 \end{array}$$

The same rule holds true for subtraction.

EXAMPLE:

Subtract 0.008 from 2.687

$$\begin{array}{r} 2.687 \\ -0.008 \\ \hline 2.679 \end{array}$$

EXERCISE 11:

Add the following:

1. 68.4
    32.1

2. 0.004
    0.026

3. 6.01
    0.023

4. (4)+(0.68) + (1.23)

5. (67.9)+(4.52)

59-19. Multiplication and Division of Decimals
When multiplying decimals, multiply as if the decimals were whole numbers. Place the decimal point in the final answer at that place, which is the sum of the decimal places in the factors, to the left of the last digit in the product.

EXAMPLE:

$$\begin{array}{r} 6.0 \\ \underline{3.0} \\ 18.00 \end{array}$$ (two places to the left of the last digit)

$$\begin{array}{r} 30.6 \\ \underline{0.007} \\ 0.2142 \end{array}$$ (four places to the left of the last digit)

$$\begin{array}{r} 9.3 \\ \underline{0.0008} \\ 0.00744 \end{array}$$ (five places to the left of the last digit)

When dividing decimals, move the decimal point in the divisor to the right of the last digit that is not zero. Also, move the decimal point in the number being divided the same number of places adding zeros as necessary. The decimal point in the answer should be placed directly above the decimal point (at its final location) in the number divided. Divide as if the numbers were whole numbers.

EXAMPLE:

$$6.8 \div 3.2 = 32 \overline{)68.000} \begin{array}{r} 2.125 \\ \hline \end{array}$$

$$\begin{array}{r} \underline{64} \\ 40 \\ \underline{32} \\ 80 \\ \underline{64} \\ 160 \\ \underline{160} \\ 0 \end{array}$$

$$16.4 \div 4.0 = 4.0 \overline{)16.4} \begin{array}{r} 4.1 \\ \hline \end{array}$$

$$\begin{array}{r} \underline{16} \\ 04 \\ \underline{4} \\ 0 \end{array}$$

EXERCISE 12:

Perform the indicated division:

1. 6/10   2. 1/6.28   3. 2.743/3.77

4. 5.372/32   5. 5.0/51   6. 0.0765/23

7. 81/0.9   8. 6/0.19   9. 48/6.254

10. 25.68/2.4

59-20. Percentage

Percentage is the process of computation in which the basis of comparison is ONE HUNDRED. Thus, 2 percent of a quantity means two parts of every hundred parts of the quantity.

The symbol of percentage is %. Percent may also be indicated by a fraction or a decimal. Thus, $5\% = \frac{5}{100} = 0.05$.

The BASE is the number on which the percentage is computed.

The RATE is the amount (in hundredths) of the base to be estimated.

The PERCENTAGE is a part or proportion of a whole expressed as so many per hundred. Percentage is the portion of the base determined by the rate.

Conversion of Decimal to Percent:

To change a decimal to percent, move the decimal point two places to the right and add the percent symbol.

EXAMPLE:

Change 0.375 to percent

Move decimal point two places to right: 37.5

Add percent symbol: 37.5%

Conversion of Fraction to Percent:

To convert a fraction to percent, divide the numerator by the denominator and convert to a decimal. Then, convert the decimal to percent.

EXAMPLE:

Change fraction $\frac{5}{8}$ to percent.

Divide numerator by denominator: $5 \div 8 = 0.625$

Convert decimal to percent: $0.625 = 62.5\%$

Thus, $\frac{5}{8} = 62.5\%$

Conversion of Percent to Decimal:

To change a percent to a decimal, omit the percent symbol and move the decimal point two places to the left.

EXAMPLE:

Change 15% to a decimal

Omit percent symbol: 15% becomes 15

Move decimal point two places to the left: 15 becomes 0.15

Thus, 15% = 0.15

EXAMPLE:

Change 110% to a decimal.

Omit percent symbol: 110% becomes 110

Move the decimal point two places to the left: 110 becomes 1.10

Thus, 110% = 1.10

Conversion of Percent to Fraction

To change a percent to a fraction, first change the percent to a decimal and then to a fraction. Reduce the fraction to its lowest terms.

EXAMPLE:

Change 25% to a fraction

Change to a decimal: 25% = 0.25

Change to a fraction: $0.25 = \dfrac{25}{100}$

Reduce fraction to lowest terms: $\dfrac{25}{100} = \dfrac{1}{4}$

EXAMPLE:

Change 37.5% to a fraction

Change to a decimal: 37.5% = 0.375

Change to a fraction: $0.375 = \dfrac{375}{1000}$

Reduce fraction to lowest terms: $\dfrac{375}{1000} = \dfrac{3}{8}$

Finding Percentage:

To find the percent of a number, write the percent as a decimal and multiply the number by this decimal. In this case, the BASE and RATE are given. The problem is to find the percentage.

EXAMPLE:

Find 5% of 140 (140 is the base, 5% is the rate, and
the product is the percentage)

5% of 140 = 0.05 x 140 = 7

EXAMPLE:

Find 5.2% of 140.

5.2% of 140 = 0.052 x 140 = 7.28

EXAMPLE:

Find 150% of 36.

150% of 36 = 1.50 x 36 = 54

EXAMPLE:

Find $\frac{1}{2}$% of 840

$\frac{1}{2}$% = 0.5%

0.5% of 840 = 0.005 x 840 = 4.20

Thus, $\frac{1}{2}$% of 840 = 4.20

In electronics, typical applications of percentage computation are used in determining tolerance values of resistors or in determining the efficiencies of motors and generators.

Finding Rate:
To find the percent one number is of another, write the problem as a fraction, change the fraction to a decimal, and write the decimal as a percent. In this case, the PERCENTAGE and BASE are given. The problem is to find the RATE.

EXAMPLE:

3 is what percent of 8? (3 is the percentage, 8 is the base, and the quotient is the rate.)

$\frac{3}{8}$ = 0.375

0.375 = 37.5% = $37\frac{1}{2}$%

Therefore, 3 is $37\frac{1}{2}$% of 8.

EXAMPLE:

What percent of 542 is 234?

$\frac{234}{542}$ = 0.4317 + (round off)

0.432 = 43.2%

Therefore, 234 is 43.2% of 542.

EXAMPLE:

125 is what percent of 50?

$$\frac{125}{50} = 2.50$$

2. 50 = 250%

Therefore, 125 is 250% of 50.

Finding Base Numbers:

To find a number when a percent of the number is known, first find 1% of the number, and then find 100% of the number. In this case, the PERCENTAGE of the number and the RATE are given. The problem is to find the BASE.

EXAMPLE:

42 is 12% of what number?

12% (base number) = 42

1% (base number) = $\frac{42}{12}$ = 3. 50

100% (base number) = 100 x 3.50 = 350

The base number is 350

Therefore, 42 is 12% of 350.

EXAMPLE:

45 is 150% of what number?

150% (base number) = 45

1% (base number) = $\frac{45}{150}$ = 0. 3

100% (base number) = 100 x 0. 3 = 30

The base number is 30.

Therefore, 45 is 150% of 30.

Expressing Accuracy of Measurements in Percent:

RELATIVE ERROR is the accuracy of a measurement expressed in percent of the total measurement. In determining the relative error, it is first necessary to establish the LIMIT OF ERROR.

The limit of error is the difference between the TRUE VALUE and the MEASURED VALUE. Assume that the reading on a scale, to the nearest tenth of an inch, is 2.2 inches. If the true value is 2. 15 inches, the limit of error is the difference between 2. 15 and 2.20, or 0.05 inch.

Relative error is computed by solving the ratio $\frac{\text{LIMIT OF ERROR}}{\text{MEASURED VALUE}}$ and expressing the result as a percent. In the scale reading above, the relative error = $\frac{0.05}{2.2}$ = 2.27%, or 2.3%.

EXERCISE 13:

Show each of the following in three forms—as a fraction or mixed number, as a decimal, and as a percent:

1. $\frac{3}{5}$ 2. 50%   3. 0.375   4. $\frac{1}{4}$

5. $62\frac{1}{2}\%$ 6. 0.6   7. $\frac{3}{10}$ 8. 70%

9. 2.25   10. $1\frac{7}{8}$ 11. 0.08   12. $\frac{3}{50}$

13. 0.18   14. $\frac{1}{4}\%$ 15. 0.025   16. 0.05

17. $8\frac{1}{3}\%$ 18. $37\frac{1}{2}\%$ 19. 105%   20. 4%

Evaluate the following:

21. 250% of 60   22. 125% of 40   23. 200% of 2   24. 225% of 400

What percent of a number is:

25. 1.5 times the number?   26. $2\frac{3}{4}$ times the number?

27. $\frac{3}{2}$ times the number?   28. $5\frac{1}{2}$ times the number?

Find the following:

29. $\frac{2}{5}\%$ of 410   30. $\frac{3}{5}\%$ of 416,000   31. $\frac{2}{5}\%$ of 85   32. 5.2% of 85

Solve the following problems:

33. Find the relative error for a limit of error of 0.05 inch in measuring 24.2 inches.

34. Find the relative error for a limit of error of 2 inches in measuring 200 yards.

Find the number when:

35. 12% of the number is 52   36. 15% of the number is 375

37. 32% of the number is 166.4   38. 8% of the number is 16

39. 84% of the number is 168   40. 17% of the number is 22

59-21. Equations and Transposition

An EQUATION is a statement of quality between two expressions. For example, x + y = 12, 3x + 5 = 20, and 3 x 9 = 27 are equations; therefore, all expressions separated by the equality are equations, whether the expressions are algebraic or arithmetical. The expression to the left of the

equality sign is called the LEFT HAND MEMBER of the equation; the expression to the right of the equality sign is called the RIGHT HAND MEMBER. Finding the values of the unknown quantities of an algebraic equation is known as solving the equation, and the answer is called the SOLUTION. If only one unknown is involved, the solution is also called the ROOT.

Solving simple equations:

1. Equal quantities may be added to both sides of an equation without changing the quality.

EXAMPLE:

Solve the equation x - 4 = 7 for x

$$x - 4 = 7$$

$$x - 4 + 4 = 7 + 4$$

$$x = 11$$

Solve the equation x - 7 = 14 for x

$$x - 7 = 14$$

$$x - 7 + 7 = 14 + 7$$

$$x = 21$$

2. Equal quantities may be subtracted from both sides of an equation without changing the quality.

EXAMPLE:

Solve the equation x + 2 = 5 for x

$$x + 2 = 5$$

$$x + 2 - 2 = 5 - 2$$

$$x = 3$$

Solve the equation x + 5 = 12 for x

$$x + 5 = 12$$

$$x + 5 - 5 = 12 - 5$$

$$x = 7$$

3. Both sides of an equation may be multiplied by the same number without changing the equality.

EXAMPLE:

Solve the equation $\frac{x}{3} = 5$ for x

$$\frac{x}{3} = 5$$

$$\frac{x}{\cancel{3}} \times \frac{\cancel{3}}{1} = 5 \times 3$$

$$x = 15$$

Solve the equation $\frac{x}{3} + \frac{x}{9} = 4$ for x

Multiply both sides of the equation by 9.

$$\frac{x}{\cancel{3}} \times \frac{\cancel{9}^{3}}{1} + \frac{x}{\cancel{9}} \times \frac{\cancel{9}}{1} = 4 \times 9$$

$$3x + x = 36$$

$$4x = 36$$

$$x = 9$$

4. Both sides of an equation may be divided by the same quantity without changing the quality.

EXAMPLE:

Solve the equation $3x = 12$ for x

$$3x = 12$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{\cancel{12}^{4}}{\cancel{3}}$$

$$x = 4$$

Solve the equation PV = RT for T

$$PV = RT$$

$$\frac{PV}{R} = \frac{\cancel{R}T}{\cancel{R}}$$

$$T = \frac{PV}{R}$$

Solving more difficult equations:

The process of adding to or subtracting from both members of an equation can be shortened by shifting a term or terms from one side of the equation to the other and changing the signs.   This operation is called TRANSPOSITION.

EXAMPLE:

Solve the equation $6x + 4 = x - 16$ for x

$$6x + 4 = x - 16$$

$$6x - x = -16 - 4$$

$$5x = -20$$

$$x = -4$$

Solve the equation $5a - 7 = 2a + 2$ for a

$$5a - 7 = 2a + 2$$

$$5a - 2a = 2 + 7$$

$$3a = 9$$

$$a = 3$$

In solving a fractional equation, first find the LCD and multiply both members of the equation, term by term; then perform the operations described previously.

EXAMPLE:

Solve the equation $\frac{x}{2} + \frac{x}{3} = 10$ for x

$$\frac{x}{2} + \frac{x}{3} = 10$$

$$\frac{3x + 2x}{6} = 10$$

$$\frac{5x}{6} = \frac{10}{1}$$

$$5x = 60$$

$$x = 12$$

Solve the equation $\frac{x - 1}{2} = 3 + x$ for x

$$\frac{x - 1}{2} = 3 + x$$

$$\frac{x - 1}{2} = \frac{3 + x}{1}$$

$$1(x - 1) = 2(3 + x)$$

$$x - 1 = 6 + 2x$$

$$x - 2x = 6 + 1$$

$$-x = 7$$

$$x = -7$$

Solve the equation $\frac{2}{x - 2} + \frac{2}{x + 4} = \frac{4}{x - 3}$ for x

$$\frac{2}{x - 2} + \frac{2}{x + 4} = \frac{4}{x - 3}$$

$$\frac{2(x + 4) + 2(x - 2)}{(x - 2)(x + 4)} = \frac{4}{x - 3}$$

$$\frac{2x + 8 + 2x - 4}{(x - 2)(x + 4)} = \frac{4}{x - 3}$$

$$\frac{4x + 4}{(x - 2)(x + 4)} = \frac{4}{x - 3}$$

$$(4x + 4)(x - 3) = 4(x - 2)(x + 4)$$

$$4x^2 - 8x - 12 = 4(x^2 + 2x - 8)$$

$$4x^2 - 8x - 12 = 4x^2 + 8x - 32$$

$$4x^2 - 4x^2 - 8x - 8x = -32 + 12$$

$$-16x = -20$$

$$16x = 20$$

$$x = \frac{20}{16} = \frac{5}{4} = 1\frac{1}{4}$$

$$x = 1\frac{1}{4}$$

Written equations:

Many practical problems are stated in words must be translated into symbols before the rules of algebra can be applied. There are no specific rules for the translation of a written problem into an equation of numbers, signs and symbols. The following general suggestions may be helpful in developing equations:

a. From the worded statement of the problem, select the unknown quantity (or one of the unknown quantities) and represent it by a letter such as X. Write the expression, stating exactly what X represents and the units in which it is measured.

b. If there is more than one unknown quantity in the problem, try to represent each unknown in terms of the first unknown.

EXAMPLE:

In simple problems, an equation may be written by an almost direct translation into algebraic symbols; thus,

Seven times a certain number diminished by 3 gives the same result as the number increased by 75

| 7 | x | Z | - | 3 | = | Z | + | 75 |

Solving the equation:

$$7Z - 3 = Z + 75$$

$$7Z - Z = 75 + 3$$

$$6Z = 78$$

$$Z = 13$$

Check: $$7(13) - 3 = 13 + 75$$

$$91 - 3 = 13 + 75$$

$$88 = 88$$

A triangle has a perimeter of 30 inches. The longest side is 7 inches longer than the shortest side and the third side is 5 inches longer than the shortest side. Find the length of the three sides.

$$\text{Let } x = \text{length of the shortest side}$$

$$x + 7 = \text{length of the longest side}$$

$$x + 5 = \text{length of the third side}$$

$$x + (x + 7) + (x + 5) = 30$$

Solving the equation:

$$x + x + 7 + x + 5 = 30$$

$$3x + 12 = 30$$

$$3x = 30 - 12$$

$$3x = 18$$

$$x = 6 - \text{shortest side}$$

$$6 + 7 = 13 \text{ longest side}$$

$$6 + 5 = 11 \text{ third side}$$

Simultaneous equations:

Simultaneous equations are two or more equations satisfied by the same sets of values of the unknown quantities. Simultaneous equations are used to solve a problem containing two or more unknown quantities. A general rule for establishing a set of simultaneous equations is that for every unknown quantity in the problem there must be an equation in the set of simultaneous equations. Thus, for two unknowns in the problem there will be two equations, three unknowns, three equations, etc.

In the solution of simultaneous linear equations three methods will be explained by the use of an example. The methods which will be used are addition, subtraction and substitution.

EXAMPLE:

Assume that the sum of two numbers is 17, and that three times the first number less two times the second number is equal to 6. What are the numbers? In setting up equations for this problem, let x equal the first number and y equal the second number. The first equation is $x + y = 17$, and the second equation is $3x - 2y = 6$.

1. Addition

$$x + y = 17$$

$$3x - 2y = 6$$

Before adding, change the y in the first equation to 2y so that the y terms drop out when added, thus, the first equation must be multiplied by 2.

$$2x + 2y = 34$$
$$\underline{3x - 2y = 6}$$
$$5x = 40$$
$$x = 8$$

Substitute x = 8 in the first equation and solve for y:

$$x + y = 17 \text{ or } 8 + y = 17$$

$$y = 17 - 8$$

$$y = 9$$

2. Subtraction

$$x + y = 17$$

$$3x - 2y = 6$$

Before subtracting, multiply the first equation by 3 so that the x terms drop out when subtracted.

$$3x + 3y = 51$$
$$\underline{3x - 2y = 6}$$
$$5y = 45$$
$$y = 9$$

Substitute $y = 9$ in the first equation and solve for y:

$$x + y = 17 \text{ or } x + 9 = 17$$

$$x = 17 - 9$$

$$x = 8$$

3. Substitution

$$x + y = 17 \text{ or } x = 17 - y$$

Substitute $x = 17 - y$ in the second equation:

$$3x - 2y = 6$$

$$3(17 - y) - 2y = 6$$

Remove the parenthesis:

$$51 - 3y - 2y = 6$$

Transpose:

$$-5y = 6 - 51$$

$$-5y = -45$$

$$5y = 45$$

$$y = 9$$

Substitute $y = 9$ in the first equation and solve for x:

$$x + y = 17 \text{ or } x + 9 = 17$$

$$x = 17 - 9$$

$$x = 8$$

Solving Formulas:

    A formula is a rule or law that states a scientific relationship. It can be expressed in an equation by using letters, symbols and constant terms.

To solve a formula, perform the same operations on both members of an equation until the desired unknown can be isolated in one member of the equation. If the numerical values for some variables are given, substitute in the formula and solve for the unknown as in any other equation.

EXAMPLE:

1. Solve the formula $T = \dfrac{12\,(D-d)}{t}$ for D.

$$T = \frac{12\,(D-d)}{t}$$

$$T = \frac{12D - 12d}{t}$$

Multiply both sides by t:

$$Tt = 12D - 12d$$

Transpose and change signs:

$$12D = Tt + 12d$$

Divide both sides by 12:

$$\frac{12D}{12} = \frac{Tt}{12} + \frac{12d}{12}$$

$$D = \frac{Tt}{12} + d$$

2. Given the formula $R_T = \dfrac{R_1\,R_2}{R_1 + R_2}$ solve for R_2.

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Cross multiply:

$$R_1 R_2 = R_1 R_T + R_2 R_T$$

Transpose and change signs:

$$R_1 R_2 - R_2 R_T = R_1 R_T$$

Factor R_2 out of the left member:

$$R_2\,(R_1 - R_T) = R_1 R_T$$

Divide both sides by $R_1 - R_T$

$$\frac{R_2\,(R_1 - R_T)}{R_1 - R_T} = \frac{R_1 R_T}{R_1 - R_T}$$

$$R_2 = \frac{R_1 R_T}{R_1 - R_T}$$

59-22. Square Roots

A FACTOR of a number is an even division of that number. As an example, the numbers 3 and 5 are even divisors of the number 15. Therefore, both 3 and 5 are factors of the number 15. In the number 4, the number 2 is a factor twice (2 x 2 = 4). The numbers in the latter example are called

EQUAL FACTORS. When a number contains only equal factors, they are called ROOTS of the number. When a number is divided into two equal factors, either of the equal factors are called SQUARE ROOT of the number.

The sign used to indicate that a square root is to be extracted is $\sqrt{}$. It is placed over the number whose square root is to be found. As an example, the square root of sixteen ($\sqrt{16}$) would mean that two equal factors of the number are to be found. The sign indicating the extraction ($\sqrt{}$) is called a RADICAL sign.

The following method is used to extract the square root of the number 401,956.

Step One:

Begin at the decimal point (which is to the right of the last digit) and divide the number into two digit groups in both directions.

$$\sqrt{\overline{40\ \ 19\ \ 56}}$$

Step Two:

Place the decimal point for the square root directly above the decimal point that appears under the radical sign.

$$\sqrt{\overline{40\ \ 19\ \ 56.}}$$

Step Three:

Determine the largest number that, when multiplied by itself, will give a product equal to or less than the first pair of digits 40. The number 6, since any number larger than 6 multiplied by itself will produce a number greater than 40. Place the 6 above the first pair of digits.

$$\frac{6\qquad\ .}{\sqrt{40\ \ 19\ \ 56.}}$$

Step Four:

Square 6 to obtain 36 and place it below the first two digits 40. Subtract 36 from 40 and obtain 4. Bring down the next pair of digits 19.

$$\frac{6\qquad\ .}{\sqrt{40\ \ 19\ \ 56.}}$$
$$\frac{36}{4\ \ 19}$$

Step Five:

Double the first digit of the answer, 6, to obtain a TRIAL DIVISOR of 12. Place the 12 to the left of 419 as shown.

$$\frac{6\qquad\ .}{\sqrt{40\ \ 19\ \ 56.}}$$
$$\frac{36}{4\ \ 19}$$
$$12$$

Step Six:

Divide the trial divisor (12) into all but the last digit of the modified remainder — 419. It will divide into 41 three times. This will be the next digit of the answer. Place the three above the second pair of digits and also place the 3 to the right of the trial divisor. Thus, the completed divisor

is 123. Multiply 123 by 3 and obtain 369. Subtract 369 from 419 to obtain 50. Bring down the next pair of digits 56.

$$
\begin{array}{r}
6 \quad 3 \quad . \\
\sqrt{40\ 19\ 56.} \\
\underline{36} \\
123 \quad\ \ 4\ 19 \\
3\ 69 \\
\underline{} \\
50\ 56
\end{array}
$$

Step Seven:

Double the first two digits of the answer 63, to obtain the new trial divisor of 126. Place the 126 to the left of 5056 as shown.

$$
\begin{array}{r}
6 \quad 3 \quad 4. \\
\sqrt{40\ 19\ 56.} \\
\underline{36} \\
123 \quad\ \ 4\ 19 \\
3\ 69 \\
126 \quad\ \ \underline{} \\
50\ 56
\end{array}
$$

Step Eight:

Divide the trial divisor 126 into all but the last digit of the modified remainder 5056. It will go into 505 four times. This will be the next digit of the answer. Place the four above the third digit, and also place the 4 to the right of the trial divisor. Thus, the completed divisor is 1264, multiply 1,264 by 4 and obtain 5056. Subtract 5056 from 5056. The remainder is zero. Therefore, the square root of 401,956 is 634.

Step Nine:

Check the answer by squaring 634.

$$634 \times 634 = 401,956$$

EXAMPLE:

Find the square root of 552.35

Step One:

Begin at the decimal point and divide the number into digit groups in both directions.

$$\sqrt{05\ 52.\ 35}$$

Step Two:

Place the decimal point for the square root directly above the decimal point that appears under the radical sign.

$$\sqrt{05\ 52.\ 35}$$

Step Three:

Determine the largest number that when multiplied by itself will give a product equal to or less than the first pair of digits, 05. The number is 2 since any number larger than 2 multiplied by itself will produce a number greater than 5. Place the 2 above the first pair of digits.

$$\overset{2\quad .}{\sqrt{05\ 52.\ 35}}$$

Step Four:

Square 2 to obtain 4 and place it below the first two digits, 05. Subtract 4 from 5 and obtain 1. Bring down the next pair of digits, 52.

$$\begin{array}{r} 2 \quad . \\ \sqrt{05\ 52.\ 35} \\ \underline{4} \\ 1\ 52 \end{array}$$

Step Five:

Double the first digit of the answer 2, to obtain a trial divisor of 4. Place the four to the left of 152 as shown.

$$\begin{array}{r} 2 \quad . \\ \sqrt{05\ 52.\ 35} \\ \underline{4} \\ 4 \quad\quad 1\ 52 \end{array}$$

Step Six:

Divide the trial divisor (4) into all but the last digit of the modified remainder 152. It will divide into 15 three times. This will be the next digit of the answer. Place the 3 above the second pair of digits and also place the three to the right of the trial divisor. Thus, the completed divisor is 43. Multiply 43 by 3 and obtain 129. Subtract 129 from 152 to obtain 23. Bring down the next pair of digits, 35.

$$\begin{array}{r} 2 \quad 3. \\ \sqrt{05\ 52.\ 35} \\ \underline{4} \\ 43 \quad\quad 1\ 52 \\ \underline{1\ 29} \\ 23 \quad 35 \end{array}$$

Step Seven:

Double the first two digits of the answer, 23, to obtain the new trial divisor of 46. Place the 46 to the left of 2335 as shown.

$$\begin{array}{r} 2 \quad 3. \\ \sqrt{05\ 52.\ 35} \\ \underline{4} \\ 43 \quad\quad 1\ 52 \\ \underline{1\ 29} \\ 46 \quad\quad 23\ 35 \end{array}$$

Step Eight:

Divide the trial divisor 46 into all but the last digit of the modified remainder, 2335. It will divide into 233 five times. This will be the next digit of the answer. Place the 5 above the pair of digits and to the right of the trial divisor. Thus, the completed divisor is 465. Multiply 465 by 5 and obtain 2325. The remainder is 10. Therefore, the square root of 552.35 is 23.5 with a remainder of 10. A more accurate answer can be obtained by adding zeros and continuing the process of extracting the square root.

$$\begin{array}{r} 2 \quad 3. \quad 5 \\ \sqrt{05\ 52.\ 35} \\ \underline{4} \\ 43 \quad\quad 1\ 52 \\ \underline{1\ 29} \\ 465 \quad\quad 23\ 35 \\ \underline{23\ 25} \\ 10 \end{array}$$

Step Nine:

Check the answer by multiplying 23.5 by itself and adding the remainder.

$$23.5 \times 23.5 = 552.25$$

$$552.25 + 0.10 = 552.35$$

Therefore, the square root of 552.35 equals 23.5 with a remainder of 10.

Square Root of a Product:

If the factors of the product are perfect squares, the square root of the product is obtained by extracting the square root of the individual factors and multiplying them together.

EXAMPLE:

$$\sqrt{64 \times 9} = \sqrt{64} \times \sqrt{9} = 8 \times 3 = 24$$

EXAMPLE:

$$\sqrt{16 \times 144} = \sqrt{16} \times \sqrt{144} = 4 \times 12 = 48$$

EXAMPLE:

$$\sqrt{169 \times 25} = \sqrt{169} \times \sqrt{25} = 13 \times 5 = 65$$

If the factors are not each a perfect square, find the product of the factors and extract the square root.

EXAMPLE:

$$\sqrt{6 \times 8} = \sqrt{48} = 6.93$$

The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

EXAMPLE:

$$\sqrt{25/169} = \frac{\sqrt{25}}{\sqrt{169}} = 5/13$$

EXAMPLE:

$$\sqrt{16/81} = \frac{\sqrt{16}}{\sqrt{81}} = 4/9$$

If the fractions numerator and denominator are not perfect squares, it would be easier to reduce the fraction to its decimal form and extract the square root.

EXAMPLE:

$$\sqrt{2/6} = \sqrt{0.333} = 0.58$$

EXERCISE 14:

Extract the square root of the following:

1. 3,969	2. 1,049.76	3. 275.56	4. 49,729
5. 13,924	6. 110,889	7. 2,735.29	8. 4,489
9. 256	10. 144,326		

Extract the square root to two decimal places.

11. 684   12. 321   13. 2   14. 5

15. 86   16. $\sqrt{16/64}$ 17. $\sqrt{3/8}$ 18. $\sqrt{64 \times 16}$

RATIO AND PROPORTION

59-23. Ratio

A knowledge of "how many" of a certain group of objects or quantities may have little meaning in a discussion, unless that quantity is compared to another quantity. For example, to say that a man has the ability to read 400 words in one minute has little meaning without comparing his rate to another. However, when his rate is compared to the 250 words per minute rate of the average reader, one can see that he is capable of a considerably higher reading rate than the average reader. To determine this comparison mathematically, his rate is divided by the average.

EXAMPLE:

$$\frac{400 \text{ words per minute}}{250 \text{ words per minute}} = \frac{400}{250} = \frac{8}{5} = 1\frac{3}{5}$$

Thus, for every 500 words read by the average reader, this man reads 8/5 or $1\frac{3}{5}$ as fast.

It is only through such comparisons that many numbers have meaning. When a relationship between two numbers is shown in this manner, they are compared as a RATIO. A ratio is a comparison of two like quantities. It is the quotient obtained by dividing the first number by the second.

For example, a gear has 40 teeth and another gear has 10 teeth. A comparison would be 40 teeth to ten teeth. This comparison could be written as a ratio in four ways: 40:10, 40÷10, 40/10, of the ratio of 40 to 10. When the emphasis is on ratio, all of these expressions would be read, "the ratio of 40 to 10".

Two quantities expressed as a ratio must be of the same kind. For example, there can be no single ratio between 12 bolts and five men. A ratio should be expressed in similar units, yards to yards, quarts to quarts, etc.

The two numbers of a ratio are called the TERMS of the ratio. The first term, or the numerator, is called the ANTECEDENT. The second term, or the denominator, is called the CONSEQUENT. In the previous example, the number 40, is the antecedent, and the number 10 is the consequent.

Since a ratio is also a fraction, all of the rules governing the operation of fractions apply to operations with ratios. Therefore, the ratios may be reduced, simplified, increased, decreased, and so forth. To reduce the terms of a ratio, such as 15 to 20, write the ratio as a fraction, and proceed as in fractions. Thus, 15 to 20 becomes:

$$\frac{15}{20} = \frac{3 \times 5}{4 \times 5} = \frac{3}{4} \cdot \frac{5}{5}$$

Since the fraction $\frac{5}{5} = 1$

Then $\frac{3}{4} \times \frac{5}{5} = \frac{3}{4}$

Therefore: $\frac{15}{20} = \frac{3}{4}$

Hence, the ratio of 15/20 is the same as the ratio 3/4.

Notice the distinction in thought between 3/4 as a fraction and 3/4 as a ratio. As a fraction, 3/4 is the single quantity "three-fourths". As a ratio, 3/4 is a comparison between two numbers, 3 and 4.

EXAMPLE:

The length of two sides of a triangle are 6 ft and 2 ft. To compare their lengths by means of a ratio, divide one number by the other and reduce to lowest terms.

$$\frac{6 \text{ ft}}{2 \text{ ft}} = \frac{3 \times 2 \times \text{ft}}{1 \times 2 \times \text{ft}} = \frac{3}{1} \times \frac{2}{2} \times \frac{\text{ft}}{\text{ft}} = \frac{3}{1} \times 1 \times 1 = \frac{3}{1}$$

The two sides of the triangle compare as 3 to 1.

59-24. Proportion
Closely allied with the study of ratio is the subject of PROPORTION. A proportion is nothing more than an equation in which the members are ratios. In other words, when two ratios are set equal to one another, a proportion is formed. When any quantity is set equal to another quantity, an equation is formed. The proportion may be written in three different ways. The fact that 15 to 20 is the same as 3 to 4 may be expressed as:

$$15:20 \ :: \ 3:4$$

$$15:20 \ = \ 3:4$$

$$\frac{15}{20} = \frac{3}{4}$$

The last two forms in this example are the most common. They may be read as "15 is to 20 as 3 is to 4". In other words, 15 has the same ratio of 20 as 3 has to 4.

One reason proportions are so important is that if any of three of the terms are given, the fourth may be found by a simple equation. In science, many chemical and physical relationships are expressed as proportions. Consequently, a familiarity with proportions will provide one method of solving applied problems. It is evident from the last form shown, 15/20 equals 3/4, that a proportion is really a fractional equation, and as such all rules for fractional equations apply.

Certain names have been given to the terms of the two ratios that make up a proportion. In a proportion the first and the last terms are called the EXTREMES. In other words the antecedent of the first ratio and the consequent of the second are called the extremes. The second and third terms are called the MEANS. The means are the consequent of the first ratio and the antecedent of the second. Summarizing,

$$5:4 \ = \ 15:12$$

                means
               extremes

It is often advantageous to change the form of a proportion. There are several rules for changing or combining the terms of a proportion without altering the equality between the members. These rules are simplifications of proven fundamental rules for arithmetical and algebraic equations. Emphasis is placed upon these rules because they are used constantly and it is in the students best interest to memorize them.

RULE No. 1. In any proportion the product of the means is equal to the product of the extremes.

EXAMPLE:

$$\frac{2}{3} = \frac{6}{9} \quad \text{therefore } 3 \times 6 = 2 \times 9$$

EXAMPLE:

$$\frac{a}{b} = \frac{c}{d} \quad \text{therefore } bc = ad$$

RULE No. 2.   The product of the means divided by either extreme gives the other extreme.

EXAMPLE:

$$\frac{2}{3} = \frac{6}{9} \; ; \quad \frac{3 \times 6}{9} = 2 \text{ or } \frac{3 \times 6}{2} = 9$$

EXAMPLE:

$$\frac{a}{b} = \frac{c}{d} \; ; \quad \frac{bc}{d} = a \text{ or } \frac{bc}{a} = d$$

RULE No. 3.   The product of the extremes divided by either mean gives the other mean.

EXAMPLE:

$$\frac{2}{3} = \frac{6}{9} \; ; \quad \frac{2 \times 9}{3} = 6 \text{ or } \frac{2 \times 9}{6} = 3$$

EXAMPLE:

$$\frac{a}{b} = \frac{c}{d} \; ; \quad \frac{ad}{b} = c \text{ or } \frac{ad}{c} = b$$

Solving for the Unknown Term:
    Using the rules of proportion solve the below proportions for the unknown term.

EXAMPLE:

$$\frac{3}{4} = \frac{9}{y}$$

Rule 2 says

$$\frac{4 \times 9}{3} = y$$

Therefore,

$$y = \frac{36}{3} = 12; \qquad \frac{3}{4} = \frac{9}{12}$$

EXAMPLE:

$$\frac{Y}{6} = \frac{12}{18}$$

Rule 3 says

$$\frac{6 \times 12}{18} = Y$$

Therefore;

$$Y = \frac{72}{18} = 4; \qquad \frac{4}{6} = \frac{12}{18}$$

EXERCISE 15:

Work the problems below and check your answers.

1. $\dfrac{6}{x} = \dfrac{18}{3}$

2. $\dfrac{3}{4} = \dfrac{x}{8}$

3. $\dfrac{x}{5} = \dfrac{8}{20}$

4. $\dfrac{6 \text{ ft}}{3 \text{ ft}} = \dfrac{x}{4 \text{ min}}$

5. $\dfrac{60 \text{ miles}}{120 \text{ miles}} = \dfrac{180 \text{ min}}{x}$

A knowledge of the properties of a proportion often provides a quick and easy method of solving word problems. However, when setting up proportion problems, be sure that the ratios are stated correctly. The ratios must be compared in the same order. In other words, if one ratio is compared lesser to greater then the other ratio must be compared in the same order. Therefore, you must analyze the problem and decide whether the unknown quantity will be lesser or greater than the known quantity of the ratio in which it occurs. The following examples will show the processes involved in setting up and solving ratio and proportion problems.

EXAMPLE:

If an automobile runs 60 miles on 6 gallons of gas, how many miles will it run on 20 gallons?

$$\frac{\text{LESSER}}{\text{GREATER}} = \frac{\text{LESSER}}{\text{GREATER}}$$

It is known that 6 gallons of gasoline is less than 20 gallons. If the automobile travels 60 miles on the 6 gallons of gasoline, how far will it travel on 20 gallons?

Let Z equal the unknown.

$$\left(\frac{\text{Lesser}}{\text{Greater}}\right) \frac{60 \text{ miles}}{Z} = \frac{6}{20} \left(\frac{\text{Lesser}}{\text{Greater}}\right)$$

Using Rule 2:

$$Z = \frac{20 \times 60 \text{ miles}}{6}$$

$$Z = \frac{20 \times 10 \text{ miles}}{1}$$

$$Z = 20 \times 10 \text{ miles}$$

$$Z = 200 \text{ miles}$$

EXAMPLE:

A pulley 60 inches in diameter is turning at a speed of 600 revolutions per minute. This pulley is connected to another pulley with a diameter of 30 inches. What is the revolutions per minute of the smaller pulley?

$$\frac{\text{LESSER}}{\text{GREATER}} = \frac{\text{LESSER}}{\text{GREATER}}$$

Ratios must be expressed between quantities of the same kind.

Let the RPM of the smaller pulley be represented by X. Analyze the problem; one ratio will be between inches and the other between revolution per minute (RPM). Also note that the smaller pulley will make more revolutions per minute than the larger one. Therefore, the answer will have to be larger than 300. Arrange the ratios in the order lesser to greater:

$$\text{Ratio of inches} \quad \frac{30 \text{ inches}}{60 \text{ inches}} = \frac{30}{60}$$

$$\text{Ratio of RPM} \quad \frac{600 \text{ RPM}}{X}$$

The proportion:

$$\text{extreme} \longrightarrow \frac{30}{60} \overset{\text{mean}}{\underset{\text{mean}}{}} \qquad \frac{600 \text{ RPM}}{X} \overset{\text{mean}}{\underset{\text{extreme}}{}}$$

Using the rule which states that the product of the means divided by either extremes equals the other extreme.

$$\frac{60 \times 600 \text{ RPM}}{30} = X$$

$$\frac{2 \times 1 \times 600 \text{ RPM}}{1} = X$$

$$2 \times 600 \text{ RPM} = X$$

$$1200 \text{ RPM} = X$$

The proportion above is called an INVERSE proportion, because the smaller the diameter of the pulley the faster it will rotate.

Two numbers are inversely proportional when an increase in one causes the other to decrease, or a decrease in one causes an increase in the other.

EXERCISE 16:

In each of the following problems, set up the correct proportions and solve for the unknown value.

1. Find the fourth proportional of 6, 3, and 12 (taken in order).

2. If a mast 8 ft high casts a shadow 10 ft long, how high is a mast that casts a shadow 40 ft long?

3. The speed of two cars is in a ratio of 2 to 5. If the slower car goes 30 mph, what is the speed of the faster car?

4. If 6 seamen can empty 2 cargo spaces in a day, how many spaces can 150 seamen empty in a day?

5. If 12 typewriters cost $1,020, how much will 9 cost at the same rate?

6. How long will it take a crew of men to stack 12,000 shells, if they can stack 3,000 shells in 2.5 hours?

7. If one inch on a map represents 50 miles, how many inches on the map represent 540 miles?

8. A blueprint is in a ratio of 1 to 3 with the actual object. If the length of the object on the blueprint is 2 feet, how long is the actual object?

ALGEBRA

ALGEBRA may be thought of as an extension of arithmetic because it extends the concept of numbers. In algebra, numbers can be represented by letters of the alphabet. The letters are called LITERAL NUMBERS. Literal numbers are used to express known or unknown quantities. Literal numbers are also used to show relationships between quantities which are related through a physical law. For example, the algebraic expression $I = E/R$ shows that (I) will vary if (E) or (R) is varied. The relationship between I, E, and R is constant, but the numerical values for these letters may take on many different values.

Before attempting the use of algebra as a tool, knowledge of basic operations and definitions must be acquired. Some basic operations and definitions are listed.

59-25. Definitions and Rules
The following signs, used in algebra, have the same meaning which they have in arithmetic. These signs are +, -, $\div$, and X. They indicate addition, subtraction, division and multiplication respectively.

Order of Operations:
When there are multiplications, divisions, additions and subtractions to be performed on a group of numbers, multiplication must be performed first, division second, and then the addition and subtraction.

EXAMPLE:

$$32 \div 4 + 3 + 2 \times 4 - 6 = X$$

$$8 + 3 + 8 - 6 = X$$

$$13 = X$$

Expression:
An algebraic EXPRESSION is a group of letters and numbers.

Factor:
Whenever two or more numbers are multiplied together, they are called FACTORS. In the expression $2\pi fL$, two is a factor of $2\pi fL$. Also $2\pi f$ is a factor of $2\pi fL$.

Terms:
A TERM of an algebraic expression is the parts of the expression not separated by a plus or minus sign. In the expression $5X+6Y-3Z$, the terms are 5X, 6Y, and 3Z.

If an algebraic expression has one term it is a MONOMIAL. An expression containing two terms is called a BINOMIAL. A TRINOMIAL has three terms. An expression containing more than three terms is called a POLYNOMIAL.

Subscript:
A SUBSCRIPT is a number or letter written to the right and the bottom of another number for further identification. In the equation $X_L = 2\pi fL$. The subscript, L is next to the X. The equation is read "X sub L is equal to two pi f L".

Laws of Exponents:

An EXPONENT in a number placed to the right and above another number (the BASE) to indicate the number of times the base is to be taken as a factor.

When like bases are multiplied together, add the exponents. When like bases are divided, subtract the exponent in the divisor from the exponent in the dividend. Examples of these operations are:

$$a^2 \cdot a^3 = a^{2+3} = a^5$$

$$a^3 / a^2 = a^{3-2} = a$$

$$a^4 / a^{-2} = a^{4+2} = a^6$$

Square Root:

To extract the SQUARE ROOT of an algebraic expression, find the two equal factors of the expression. Both factors are square roots. Examples of this operation are:

$$\sqrt{a^4 b^8} = a^2 b^4$$

$$\sqrt{a^8 b^4 x^2} = a^4 b^2 x$$

Negative Numbers:

NEGATIVE NUMBERS can be considered to be numbers with a value less than zero. They are necessary if subtractions like 4-8 are to be performed. Negative numbers have a physical meaning when applied to the appropriate quantities. It is absurd to speak of a physical length which is less than zero units long, but it is quite common to speak of a temperature that is less than zero degrees. The idea of applying the appropriate numbers to physical quantities should be considered whenever working with numbers.

Real Numbers:

The REAL number system contains both POSITIVE and NEGATIVE numbers. Figure 59-1 shows the graphical representation of the real number system. On a NUMBER SCALE positive numbers are plotted to the right of zero. Negative numbers are plotted to the left of zero. The zero point is frequently called the ORIGIN.

Addition:

The two basic rules for addition are:

1. To add two or more numbers with like signs, add the magnitudes and prefix the common sign.

2. To add a positive and negative number, compute the difference between the magnitudes and prefix the sign of the larger magnitude.
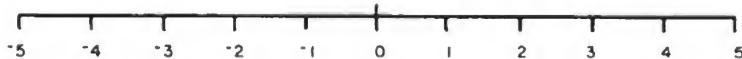
Figure 59-1 - Real numbers on the number scale.

Examples of this operation are:

$$
\begin{array}{r}
30 \\
\underline{40} \\
70
\end{array}
\qquad
\begin{array}{r}
-40 \\
\underline{20} \\
-20
\end{array}
\qquad
\begin{array}{r}
-40 \\
20 \\
\underline{-30} \\
-50
\end{array}
$$

EXERCISE 17:

Add the following:

1.	80	2.	-250	3.	-60	4.	102	5.	-86
	-30		50		-40		98		96
	60		180		-200		-60		-11

Subtraction:

The rule for subtraction is:

1. To subtract, change the sign of the subtrahend and add.

Examples of this operation are:

$$
\begin{array}{r}
40 \\
\underline{-60} \\
100
\end{array}
\qquad
\begin{array}{r}
-100 \\
\underline{60} \\
-160
\end{array}
\qquad
\begin{array}{r}
-250 \\
\underline{-75} \\
-175
\end{array}
\qquad
\begin{array}{r}
300 \\
\underline{400} \\
-100
\end{array}
$$

EXERCISE 18:

Perform the subtraction of the following:

1.	-275	2.	-415	3.	660	4.	450	5.	722
	-113		115		320		620		-131

Multiplication of Real Numbers:

The rules for multiplication are:

1. The product of two numbers that have like signs is a positive number.

2. The product of two numbers that have unlike signs is a negative number.

Examples of this operation are:

$$45 \times 30 = 1350 \qquad\qquad (-20) \times (-10) = 200 \qquad\qquad (-405) \times (2) = -810$$

EXERCISE 19:

Perform the following:

1. $(-420) \times (-30) =$ 2. $(60) \times (-30) =$ 3. $(-200)(40) =$

4. $(-31)(62) =$ 5. $(811)(-22) =$

Division of Real Numbers:

The rules for the division of real numbers are:

1. The quotient of the division of like terms is positive.

2. The quotient of the division of unlike terms is negative.

Examples of this operation are:

$50/(-10) = -5$ $650/25 = 26$ $(-40)/(-4) = 10$

EXERCISE 20:

Perform the following operations:

1. $100/(-25) =$ 2. $(-60)/(-4) =$ 3. $600/(-30) =$

4. $891/(-62) =$ 5. $322/(-12) =$

Grouping Terms:
If the additions, subtractions, multiplications and divisions are performed in a single operation; the terms of the expression are frequently grouped. GROUPING clarifies the operations. Terms are grouped using PARENTHESIS ( ); BRACKETS, [] ; BRACES, { }.

Examples of this operation are:

$$2 \left\{ [5x(6-3)] - (18/9) \right\}$$

The operations inside the inner most signs of grouping are performed first. The evaluation of the above expression is:

$$2 [(2.3) - 2] = 26$$

In removing the signs of grouping from an expression, the sign of each term within the grouping should be changed if the grouping symbol is preceded by a negative sign. The signs of the terms should not be changed if the grouping sign is preceded by a positive sign.

An example of this operation is:

$$50 - (30 + 10) + (20 - 5) - (8 \times 2)$$

To evaluate this expression, the parenthesis are removed. The expression then becomes:

$$50 - 30 - 10 + 20 - 5 - 16 = 9$$

EXERCISE 21:

Perform the following operations:

1. $5 \times (4-3) + (6-2) - (-10 + 5) =$ 2. $-(4-3) + (6+10) =$

3. $- [5 + 6 - (3+2)] =$ 4. $10 [20 + 30 - (5 + 10)] =$

5. $-6 [3 + 21 - (8 + 11) + (16 + 2) - (3 - 21)] =$

Monomials:
A MONOMIAL is an expression consisting of one term. The expressions 5a, -6b, 3X, and $4X^2$ are all classified as monomials.

Addition of Monomials:
Monomials may be added or subtracted if they are like quantities. The quantity 5a cannot be directly added to 6b unless a is similar in nature to b. If "a" indicates resistors and "b" indicates in-

ductors, it would be meaningless to talk of the sum of five resistors and six inductors. If "a" and "b" represent other numbers then the addition is easily carried out. The addition of 5a and 6b is written in this manner.

$$5a + 6b$$

This is all that may be done with this expression.

Monomials made up of like terms can be added directly. Consider the addition of 2a and 4a.

$$2a + 4a = 6a$$

In this operation, the numerical values were added directly. The numerical coefficients tell "how much", and the literal factors tell "of what". The addition of 2a and 4a can be represented by:

$$(a+a) + (a+a+a+a) = a+a+a+a+a+a$$

$$2a + \quad 4a \quad = 6a$$

EXERCISE 22:

ADD:

1. $(-6a) + (4a)$	2. $(-3X) + (-4X)$
3. $(12a^2) + (6a^2)$	4. $(2ab) + (3ba)$
5. $(16j) + (-8j)$	6. $(2I_1R_1) + (-6I_1R_1)$
7. $(20I_2R_2) + (15I_2R_2)$	8. $(3j) + (-4j)$
9. $(6I) + (3I)$	10. $(6ZY) + (-12ZY)$

Subtraction of Monomials:

To subtract a monomial A from a monomial B means to find a monomial C such that C added to A gives B.

$$6a - 4a = 2a$$

In the above equation, 4a is subtracted from 6a. To perform this subtraction a monomial was found which when added to 4a gave 6a.

Examples of this operation are:

$12a - 14a = -2a$	$-10b - 6b = -16b$

EXERCISE 23:

SUBTRACT:

1. $(-3a) - (4a)$	2. $(-6IR) - (+3IR)$
3. $(15I_2R_2) - (20I_2R_2)$	4. $(3j) - (-4j)$
5. $(30j) - (-40j)$	6. $(20I_2R_2) - (10I_2R_2)$
7. $(2T_1R_1) - (-6I_1R_1)$	8. $(16W^2) - (8W^2)$
9. $(-16XY) - (-12XY)$	10. $(-32ZY) - (-4ZY)$

Multiplication of Monomials:

   When monomials are multiplied, the coefficients (numerical values) are multiplied algebraically. Literal numbers are multiplied using the laws of exponents.

Examples of this operation are:

$$15a \cdot 4a = 60a^2 \qquad\qquad -5a \cdot 3a = -15a^2$$

$$(-6b)(-3b) = 18b^2 \qquad\qquad 4a(-3b) = -12ab$$

EXERCISE 24:

   Perform the indicated operations:

   1. $(3X^3)(4X^4)$ 2. $(-3ab^3)(3a^3b)$

   3. $(-4X^3Y)(-3XY^3)$ 4. $(2^x)(2Y)$

   5. $(3I)(4I)$ 6. $(15X^1)(6X)$

   7. $(6j)(4j)$ 8. $(30j)(-40j)$

   9. $(-15j)(-20j)$ 10. $(-j60)(-j30)$

Division of Monomials:

   In the division of monomials, the rule of exponents is applied to the literal factors. The numerical coefficients are divided the same as the other real numbers. Literal numbers are divided using the laws of exponents.

Examples of this operation are:

$$16Z/4Z = 4 \qquad\qquad 18W^2/6W = 3W$$

$$-15Y^2/3Y = -5Y \qquad\qquad -40X^2/-10X = 4X$$

EXERCISE 25:

1. $\dfrac{X^4}{X^2}$ 2. $\dfrac{fR}{2\ fL}$ 3. $\dfrac{a^9b^4}{b^6b^3}$ 4. $\dfrac{a^2b^2}{ab^2}$

5. $\dfrac{5a^4b}{10a^2b^3}$ 6. $\dfrac{10X^2Y^3Z^4}{-5XY^2Z^3}$ 7. $\dfrac{a^2bc^2}{abc}$ 8. $\dfrac{2\ f}{4\ fL}$

9. $\dfrac{16a^2b}{4a^2b}$ 10. $\dfrac{16IR}{4IR}$

Binomials:

   A BINOMIAL is an expression made up of two terms. This means that there is an indicated addition or subtraction of two monomials shown below.

$$5a + 6b, \qquad\qquad 3x + 4y, \qquad\qquad 6Z - 7c$$

Addition of Binomials with Identical Literal Factors:

To add two or more binomials, add the numerical coefficients of like factors.

Examples of this operation are:

$$
\begin{array}{ccc}
5X - 6y & -10Z - 4X & -4X - 4Y \\
3X - 4y & 3Z - 3X & 2X - 3Y \\
\hline
8X - 10y & -7Z - 7X & -2X - 7Y
\end{array}
$$

Subtraction of Binomials with Identical Literal Factors:

To subtract a binomial B from a binomial A change the sign of the terms of B and add.

Examples of this operation are:

$$
\begin{array}{ccc}
10X + 4Y & 30X - 60Z & -35a - 40b \\
5X - 8Y & -20X - 40Z & 80a + 60b \\
\hline
5X + 12Y & 50X - 20Z & -115a - 100b
\end{array}
$$

EXERCISE 26:

Perform the indicated operations:

1. $(30 + j40) + (30 - j40)$ 2. $(6a + 4b) + (8a - 2b)$

3. $(9 - j12) - (3 + j4)$ 4. $(3a + 4b) - (6a - b)$

5. $(6XY + 4Z) - (-3XY - 4Z)$ 6. $(6XY - 4Z) - (-3Z + 4XY)$

7. $(2a + B) + (3a + 5b)$ 8. $(X + 6) - (3X + 7)$

9. $(4a^2 - b) - (2a^2 + b)$ 10. $(6X^2Y^2 - 5Z^2) - (6X^2 + 5Z^2)$

Multiplication of Binomials:

To multiply a binomial A by a binomial B, multiply each term of A by each term of B. Then combine or simplify the results.

Examples of this operation are:

$$
\begin{array}{l}
5a + 8b \\
3a + 6b \\
\hline
15a^2 + 24ab \\
\quad\quad + 30ab + 48b^2 \\
\hline
15a^2 + 54ab + 48b^2
\end{array}
$$

$$
\begin{array}{l}
-10X + 20Y \\
3X + 4Y \\
\hline
-30X^2 + 60XY \\
\quad\quad - 40XY + 80Y^2 \\
\hline
-30X^2 + 20XY + 80Y^2
\end{array}
$$

EXERCISE 27:

Multiply and add like terms:

1. $(X+3)(X-4)$ 2. $(6a+3b)(7a-4b)$

3.   (2K+C)(3K+2b)   4.   (2+b)(2+b)

5.   (ax+y)(ax+y)   6.   (30+j 40)(3-j 4)

7.   (16+j 10)(20-j 40)   8.   (50-j 40)(60+j 20)

9.   (-j60)(60+j 60)   10.   30(20-j 30)

Division of a Polynomial by a Polynomial:
  The following steps should be taken when dividing a polynomial by another polynomial.

1.   Arrange the divisor and dividend in ascending or descending powers of a common literal factor.

2.   Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient. Multiply all terms of the divisor by the quotient. Place these terms under the dividend. Terms of this product should be placed under like terms of the dividend and sub-tracted.

3.   The remainder is considered the new dividend, and step 2 is repeated until the remainder is too small to be divided by the divisor.

Examples of this operation are:

$$
\begin{array}{r}
x + 2 \\
x + 1 \overline{\smash{\big)}\ x^2 + 3x + 2} \\
\underline{x^2 + x} \\
2x + 2 \\
\underline{2x + 2} \\
0
\end{array}
$$

$$
\begin{array}{r}
6x^2 + 5x - 9 \\
x^2 + 2 \overline{\smash{\big)}\ 6x^4 + 5x^3 + 3x^2 + 4} \\
\underline{6x^4 + 12x^2} \\
5x^3 - 9x^2 + 4 \\
\underline{5x^3 + 10x} \\
- 9x^2 - 10x + 4 \\
\underline{- 9x^2 - 18} \\
\text{remainder} = \quad - 10x + 22
\end{array}
$$

Square Root:
  Some basic rules to be observed in extracting the square root of a number are:

1.   The square root of a product is equal to the product of the square root.

Examples of this operation are:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{200} = \sqrt{2 \cdot 100} = \sqrt{2} \times \sqrt{100} = \sqrt{2} \times 10 = 14.14$$

$$\sqrt{4\pi^2 LC} = \sqrt{4\pi^2} \cdot \sqrt{LC} = 2\pi \cdot \sqrt{LC}$$

2.   The square roots of a ratio (division) is equal to the ratio of the square roots.

Examples of this operation are:

$$\sqrt{a/b} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{4/9} = \frac{\sqrt{4}}{\sqrt{9}} = 2/3$$

3. The square root of a sum is equal to the square root of that sum.

Examples of this operation are:

$$\sqrt{a + b} = \sqrt{a + b}$$

$$\sqrt{10 + 15} = \sqrt{25} = 5$$

The square root of ten plus fifteen is not equal to the square root of ten plus the square root of fifteen.

4. The square root of an indicated subtraction is equal to the square roots of that subtraction.

Examples of this operation are:

$$\sqrt{a - b} = \sqrt{a - b}$$

$$\sqrt{110 - 10} = \sqrt{100} = 10$$

The square root of one hundred and ten is not equal to the square root of one hundred minus the square root of ten.

Factoring a Monomial from a Polynomial:

Frequently in performing computations with polynomials, it is convenient to FACTOR one of the expressions. Factoring may aid in reducing the result of the computations to the simplest form.

Examples of this operation are:

$$\frac{5a + 6a^2 b}{a} = \frac{a(5 + 6ab)}{a} = 5 + 6ab$$

Since a is common to both terms of the numerator, it can be factored out. The solution of the problem is shown above in its simplest form.

EXERCISE 28:

Find the roots of the following problems.

1. $\sqrt{9x^4 y^6}$ 2. $\sqrt{16y^2 z^4}$ 3. $\sqrt{4a^2 b^2}$ 4. $\sqrt{64a^6 b^5}$

5. $\sqrt{\dfrac{64}{4}}$ 6. $\sqrt{64 + 16}$ 7. $\sqrt{144 - 64}$ 8. $\sqrt{16x^2 + 8x^2}$

9. $\sqrt{\dfrac{16x^2}{4x^2}}$ 10. $\sqrt{15^2 + 8^2}$

59-26. Scientific Notation (Powers of Ten)

The technique of using powers of 10 can greatly simplify mathematical calculations. A number containing many zeros to the right or to the left of the decimal point can be delt with much more readily when put in the form of powers of 10. For example $0.0000037 \times 0.000021$ can be handled more easily when put in the form $3.7 \times 10^{-6} \times 2.1 \times 10^{-5}$.

Table of Powers of 10: The table below gives some of the values of the powers of 10.

Number	Powers of 10	Number	Powers of 10
0.000001	10^{-6}	1	10^{0}
0.00001	10^{-5}	10	10^{1}
0.0001	10^{-4}	100	10^{2}
0.001	10^{-3}	1000	10^{3}
0.01	10^{-2}	10000	10^{4}
0.1	10^{-1}	100000	10^{5}
		1000000	10^{6}

Expressing numbers in Scientific Notation: Any number written as the product of an integral power of 10 and a number between 1 and 10 is said to be expressed in SCIENTIFIC NOTATION.

The following rules are set down as an aid in expressing large numbers as numbers between 1 and 10 times a power of 10. The rules are stated and examples are given for converting large whole numbers or decimals into scientific notation.

Rule:

In expressing a large whole number as a smaller number usually between 1 and ten, place a decimal point to the right of the last figure of the number and move it to the left until a number to the left of the decimal point is between one and 10. The number of places the decimal point was moved will give the proper positive power of 10.

EXAMPLE:

$$637 = 6.37 \times 10^{2}$$
$$2,700 = 2.7 \times 10^{3}$$
$$56.33 = 5.633 \times 10^{1}$$

$$9,628,000 = 9.628 \times 10^{6}$$
$$5,622.8 = 5.6228 \times 10^{3}$$
$$873,000 = 8.73 \times 10^{5}$$

Rule:

In expressing a decimal as a whole number between 1 and 10, move the decimal point to the right until there is a number between 1 and 10. Count the number of places the decimal point was moved and this will be the proper negative power of 10.

EXAMPLE:

$$0.871 = 8.71 \times 10^{-1}$$
$$0.0021 = 2.1 \times 10^{-3}$$
$$0.00000017 = 1.7 \times 10^{-7}$$

$$0.00078 = 7.8 \times 10^{-4}$$
$$0.063 = 6.3 \times 10^{-2}$$
$$0.000029 = 2.9 \times 10^{-5}$$

Addition and subtraction of numbers in scientific notation: Numbers expressed in scientific notation can only be added or subtracted if the powers of 10 are the same. For example, 3×10^{5} can be added to 2×10^{5} to get 5×10^{5}; however, 3×10^{6} cannot be added to 2×10^{5} because the powers of 10 are not the same. The number 3×10^{6} can be changed to 30×10^{5}, however, and it can then be added to 2×10^{5} to obtain 32×10^{5}. The answers to problems solved by using scientific notation can be left in the exponential form. In the examples below, however, the answers are converted to the decimal form to aid in understanding this technique.

EXAMPLES:

Add 450,000 and 763,000
$$450,000 + 763,000 = 4.5 \times 10^{5} + 7.63 \times 10^{5}$$
$$= 12.13 \times 10^{5} = 1.213 \times 10^{6}$$
$$= 1,213,000$$

Add 0.00006825 and 0.00000754
$$0.00006825 + 0.00000754 = 68.25 \times 10^{-6} + 7.54 \times 10^{-6}$$
$$= 75.79 \times 10^{-6} = 7.579 \times 10^{-5}$$
$$= 0.00007579$$

Subtract 0.00000433 from 0.00005

$$0.00005 - 0.00000433 = 50 \times 10^{-6} - 4.33 \times 10^{-6}$$
$$= 45.67 \times 10^{-6} = 4.567 \times 10^{-5}$$
$$= 0.00004567$$

Multiplication of numbers in scientific notation:

When multiplying numbers written in scientific notation the low of exponents referring to multiplication of numbers raised to a power is applicable. Expressed in general form:

$$A^m \cdot A^n = A^{m+n} \ (A \neq 0)$$

EXAMPLE:

Multiply 100,000 by 1,000
$$100,000 \times 1,000 = 10^5 \times 10^3 = 10^{5+3} = 10^8 = 100,000,000$$

Multiply 25,000 by 5,000
$$25,000 \times 5,000 = 2.5 \times 10^4 \times 5 \times 10^3 = 2.5 \times 5 \times 10^{4+3}$$
$$= 12.5 \times 10^7 = 1.25 \times 10^8$$
$$= 125,000,000$$

Multiply 1,800, 0.000015, 300 and 0.0048
$$1,800 \times 0.000015 \times 300 \times 0.0048$$
$$= 1.8 \times 10^3 \times 1.5 \times 10^{-5} \times 3 \times 10^2 \times 4.8 \times 10^{-3}$$
$$= 1.8 \times 1.5 \times 3 \times 4.8 \times 10^{3-5+2-3}$$
$$= 38.88 \times 10^{-3} = 3.888 \times 10^{-2}$$
$$= 0.03888$$

Division of numbers in scientific notation: When dividing numbers written in scientific notation the law of exponents referring to the division of numbers raised to a power. Expressed in general form:

$$\frac{A^m}{A^n} = A^{m-n} \quad (a \neq 0)$$

EXAMPLE:

Divide 14,400,000 by 1,200,000

$$\frac{14,400,000}{1,200,000} = \frac{144 \times 10^5}{12 \times 10^5} = \frac{144}{12} \times 10^{5-5} = 12$$

Divide 75,000 by 0.0005

$$\frac{75,000}{0.0005} = \frac{75 \times 10^3}{5 \times 10^{-4}} = \frac{75}{5} \times 10^{3+4} = 15 \times 10^7$$

$$= 150,000,000$$

Divide 98,100 by 0.0025, 180 and 1,090,000

$$\frac{98,100}{0.0025 \times 180 \times 1,090,000}$$

$$= \frac{9.81 \times 10^4}{2.5 \times 10^{-3} \times 1.8 \times 10^2 \times 1.09 \times 10^6}$$

$$= \frac{9.81 \times 10^4}{2.5 \times 1.8 \times 1.09 \times 10^{-3+2+6}}$$

$$= \frac{9.81 \times 10^4}{4.905 \times 10^5} = 2 \times 10^{-1}$$

$$= 0.2$$

Finding the power of a number in scientific notation: When finding the power of a power which is raising to a power a number expressed in scientific notation to a power the applicable low of exponents is used:

$$(A^m)^n = A^{m \times n} \quad (A \neq 0)$$

EXAMPLE:

Square 15,000
$$(15,000)^2 = (15 \times 10^3)^2 = 15^2 \times 10^{3 \times 2}$$
$$= 225 \times 10^6 = 2.25 \times 10^8$$
$$= 225,000,000$$

Cube 2,000
$$(2,000)^3 = (2 \times 10^3)^3 = 2^3 \times 10^{3 \times 3}$$
$$= 8 \times 10^9$$
$$= 8,000,000,000$$

Square 0.0000075
$$(0.0000075)^2 = (7.5 \times 10^{-6})^2 = 7.5^2 \times 10^{-6 \times 2}$$
$$= 56.25 \times 10^{-12} = 5.625 \times 10^{-11}$$
$$= 0.00000000005625$$

Finding the root of a number in scientific notation: When finding the root of a number raised to a power which is finding the root of a number expressed in scientific notation the applicable low of exponents is used:

$$\sqrt[n]{A^m} = A^{m/n} \quad (A \neq 0)$$

EXAMPLE:

Find the square root of 25,000,000

$$\sqrt{25,000,000} = \sqrt{25 \times 10^6} = \sqrt{25} \times \sqrt{10^6}$$
$$= \sqrt{25} \times 10^{6/2} = 5 \times 10^3$$
$$= 5,000$$

Find the square root of 144,000,000

$$\sqrt{144,000,000} = \sqrt{144 \times 10^6} = \sqrt{144} \times \sqrt{10^6}$$
$$= 12 \times 10^{6/2} = 12 \times 10^3$$
$$= 12,000$$

Find the cube root of 0.000008

$$\sqrt[3]{0.000008} = \sqrt[3]{8 \times 10^{-6}} = \sqrt[3]{8} \times \sqrt[3]{10^{-6}}$$
$$= 2 \times 10^{-6/3} = 2 \times 10^{-2}$$
$$= 0.02$$

COMMON LOGARITHMS

Until the 17th century, people who used mathematics in their work were constantly faced with the necessity of laboriously carrying out their computations in a manner similar to the operations previously described in this chapter. However, in the 17th century, through the contributions of men

such as Napier, Briggs, and others; a system called LOGARITHMS based on the use of exponential laws was developed. With this system, the number of mathematical computations were reduced. For example, in astronomy, the use of logarithms reduced to a few days the computation time that previously required months to perform.

59-27. Base Ten

The word "logarithm" is derived from two Greek words, LOGAS and ARITHMOS which mean, respectively, "proportion" and "number". This combination of words was selected because of the way through which the first table of logarithms came into being. By the use of logarithms, the process of multiplication is reduced to simple addition, division is reduced to subtraction, raising a number to a power is reduced to simple multiplication, and extracting a root is reduced to simple division.

A LOGARITHM IS AN EXPONENT. In the example, $10^2 = 100$, it will be recalled that the number 10 is called the base, 100 is the power, and the number 2 is the exponent. The exponent, 2, in this application may also be called a logarithm of the number 100 in the base 10 system. Specifically, the logarithm of a number to a given base is the exponent by which the base must be raised to yield that number. In the previous example, the exponent of the base 10 is the number 2. The logarithm of the number 100 must therefore be the number 2 when the base of the system is assumed to be 10.

Written as a logarithm:

$$10^2 = 100 \quad \text{is} \quad \log_{10} 100 = 2$$

Thus, by definition, the logarithm and the exponent are the same number. This name "logarithm" for an exponent was developed to denote a special application of the laws of exponents.

Any number may be used as a base for a system of logarithms. The selection of a base is a matter of convenience. Briggs, the originator of the common logarithm system presently used, in 1617, found that the base ten possessed many advantages not obtainable in ordinary calculations with other bases. The selection of 10 as a base proved so satisfactory that it is used almost exclusively in ordinary calculations. Logarithms with a base ten are called COMMON LOGARITHMS.

When the number 10 is used as a base, it is not necessary to so indicate. It is often understood to be 10 when no other base is given. As an example, instead of writing $\log_{10} 100 = 2$, the expression, $\log 100 = 2$ is satisfactory.

The following table illustrates the relationship between the powers of 10 and the logarithms of certain numbers.

Exponential Form	Logarithmic Form
$10^3 = 1000$	$\log 1000 = 3$
$10^2 = 100$	$\log 100 = 2$
$10^1 = 10$	$\log 10 = 1$
$10^0 = 1$	$\log 1 = 0$
$10^{-1} = 0.1$	$\log 0.1 = -1$
$10^{-2} = 0.01$	$\log 0.01 = -2$
$10^{-3} = 0.001$	$\log 0.001 = -3$

TABLE 59-1. Typical Conversions from Exponential
Form to Logarithmic Form

Notice in the preceding table that only numbers which can be represented as a power of ten have whole numbers for logarithms. Also notice that the log of any number between 100 and 1,000 would be between the numbers 2 and 3. That is, the logarithm of a number between 100 and 1,000 would be the number 2 plus some decimal quantity.

EXAMPLE:

The log of the number 67 would be 1 plus some decimal value.

Because:

Exponential Form	Logarithmic Form
$10^1 = 10$	$\log 10 = 1$
$10^{1+x} = 67$	$\log 67 = 1+x$
$10^2 = 100$	$\log 100 = 2$

Log 10 equals 1 and the log of 100 equals 2. Therefore, the log 67 (a number between 10 and 100) would be between the numbers 1 and 2, or the number 1 plus a decimal.

To represent the logarithm of any number, it is necessary to utilize decimal powers.

EXAMPLE:

$$10^{2.5563} = 360$$

$$\log 360 = 2.5563$$

Likewise:

$$10^{2.5224} = 333$$

$$\log 333 = 2.5224$$

Using the same logic, it follows that the logarithm of a number between 0.1 and 0.01 would be between -1 and -2.

EXAMPLE:

$\log 0.03 =$

Exponential Form	Logarithmic Form
$10^{-1} = 0.1$	$\log 0.1 = -1$
$10^{-2+x} = 0.03$	$\log 0.03 = -1+x$
$10^{-2} = 0.01$	$\log 0.01 = -2$

Every logarithm is composed of two parts. The whole part is called a CHARACTERISTIC. The characteristic will always be positive for any number greater than one, and negative for any number less than one.

The decimal portion of the logarithm is called the MANTISSA and by convention is always kept positive.

The characteristic of a number being converted to a logarithm may be determined by inspection. It may be done in the following manner:

Place the number to be expressed as a logarithm in standard form (scientific notation). The characteristic of the number will be equal to the exponent that the number 10 is raised to.

EXAMPLE:

What is the characteristic of the number 684?

Expressing the number in standard form:

$$684 = 6.84 \times 10^2$$

The exponent, 2, is the characteristic of the number 684.

EXAMPLE:

What is the characteristic of the number 0.0684?

Expressing the number in standard form:

$$0.0684 = 6.84 \times 10^{-2}$$

Therefore, the characteristic of the number 0.0684 is -2.

If the characteristic is negative, it is customary to place the negative sign above the number which is the characteristic. Using the previous example of (-2), the characteristic would be written $\bar{2}$.

EXAMPLE:

$$\log 0.0461 = \log 4.61 \times 10^{-2} = \bar{2}.6637$$

The $\bar{2}.6637$ means -2 + 0.6637. If the minus sign precedes both the characteristic and the mantissa, it would tend to indicate that both the characteristic and the mantissa are negative. This of course would be incorrect because the mantissa must be positive. Therefore, the confusion regarding the sign is eliminated by placing the sign above the characteristic.

Another method used to indicate a negative characteristic is to add a positive number (usually ten) to the characteristic and subtract the same number from the mantissa. If the absolute value of the characteristic is greater than ten, the number chosen to be added to the characteristic and subtracted from the mantissa is the next higher multiple of ten.

EXAMPLE:

$$\log 0.046 = \bar{2} \text{ plus } 0.6637$$

May be written:

$$10 - 2 + 0.6637 - 10$$

Simplifying:

$$8.6637 - 10$$

EXERCISE 28:

Find the characteristics of the following numbers.

1. 609	2. 0.004	3. 6,832	4. 60,333	5. 0.0037
6. 4,000	7. 0.5064	8. 22.34	9. 687.3	10. 406,000
11. 0.243	12. 0.0205	13. 634,000	14. 2.64	15. 0.00004

Since the characteristic of the logarithm of a number may be found by inspection, it is necessary to calculate only the mantissa of the logarithms. Mantissas can be derived by use of advanced math-

ematics. However for convenience, the decimal part of the logarithm has been computed and placed in tables. To find the logarithm of a number.

1. Place the number in standard form.

2. Write the characteristic before locating the mantissa in the table.

3. Find the mantissa which corresponds to the significant figures of the number in the log table.

4. Add the characteristic and mantissa to produce the logarithm of the number.

N	0	1	2	3	4	5	6	7	8	9
21	.3222	.3243	.3263	.3284	.3304	.3324	.3345	.3365	.3385	.3404
(22)	.3424	.3444	.3464	.3483	.3502	(.3522)	.3541	.3560	.3579	.3598
(23)	.3617	.3636	(.3655)	.3674	.3692	.3711	.3729	.3747	.3766	.3784
(24)	.3802	.3820	.3838	.3856	(.3874)	.3892	.3909	.3927	(.3945)	.3962
25	.3979	.3997	.4014	.4031	.4048	.4065	.4082	.4099	.4116	.4133

TABLE 59-2. Sample Logarithmic Table

Table 59-2 is a section of a complete table of common logarithms and lists some typical numbers with their associated mantissas. A complete table of common logarithms is included at the end of this volume.

EXAMPLE:

Find the log of the number 232.

$$232 = 2.32 \times 10^2$$

The exponent 2 is the characteristic. The significant figures are 232. The first two significant figures are found under the column marked N in the log tables. The third significant figure, 2, is found in the fourth column from the left. The mantissa is the decimal number in the row containing 23, and in the column under the number 2. The mantissa of the significant figures 232 is .3655. Therefore, the log 232 equals the characteristic, 2, plus the mantissa, .3655; or 2 + .3655.

EXAMPLE:

Find the log of 0.00248.

$$0.00248 = 2.48 \times 10^{-3}$$

Therefore, $\overline{3}$ is the characteristic. The significant figures are 248. The first two significant figures are found in the column marked N in the log tables. The third significant figure, 8, is located in the tenth column from the left. The mantissa is the decimal number in the row containing 24, and in the column under 8. The mantissa of the significant figures 248 is .3945. Therefore, the log of 0.00248 is -3 + .3945 or $\overline{3}$.3945, or 7.3945 -10.

EXERCISE 29:

Find the logarithms of the following numbers.

1. 681   2. 0.00382   3. 0.004 x 10   4. 682 x 10   5. 0.043

6. 0.004   7. 1.95   8. 219   9. 31.6   10. 2.81 x 10

Finding a Number Corresponding to a Given Logarithm

In most problems involving logarithms, it is necessary to find not only the logarithm of numbers; but also to use the inverse process to find the number corresponding to a logarithm — the ANTILOG-ARITHM. The word "antilogarithm" is abbreviated antilog.

When a mantissa of a logarithm is given exactly in the table, finding the antilog is relatively simple.

EXAMPLE:

What number has 3.3874 for its logarithm?

The process of determining the antilog is as follows:

1. Find the mantissa in the tables. Remember, the characteristic is not part of the mantissa.

2. Find the significant figures that correspond to the mantissa .3874. Write the significant figures as a number between 1 and 10 (244 is the significant figure). It should be written as 2.44.

3. Since the characteristic is 3, the significant figures will be multiplied by 10 raised to the third power.

4. Therefore, antilog 3.3874 = 2.44 x 10^3, or equal to 2,440.

EXAMPLE:

Find the number which has 2.3522 for its logarithm.

The mantissa corresponds to a number equal to 2.25 as found in the log tables. Since the characteristic is the number 2, the number will be written as:

$$2.25 \times 10^2 = 225$$

Therefore, the logarithm of the number 225 is 2.3522.

When using the Logarithm Tables to find antilogarithms there are occasions when the mantissa falls between two numbers. If the number is exactly one half or more of the difference between the two numbers use the next higher number. If the number is less than half use the lower number.

EXERCISE 30:

Find the antilogarithms of the following numbers.

1. 3.5224	2. $\overline{3}$.6964	3. 8.6117 -10	4. 4.6721
5. 3.8129	6. 4.3610	7. $\overline{6}$.8513	8. 4.0755 -10
9. 1.4298	10. 5.5513	11. $\overline{1}$.6618	12. 3.7067
13. 6.6893 -10	14. 14.7059 -20	15. 8.3365 -20	

59-28. Addition and Subtraction of Logarithms

Logarithms are added and subtracted arithmetically. However, the mantissa must be kept positive. Therefore, every negative characteristic should be expressed as a positive characteristic added to a negative number.

EXAMPLE:

Add the logarithms 4.3010 and $\overline{6}$.8513.

The negative characteristic must be changed to a positive characteristic before the addition can be affected.

Therefore:

$$\overline{6}.8513 = 4.8513 - 10$$

Completing the addition arithmetically:

$$\begin{array}{r} 4.3010 \\ 4.8513 - 10 \\ \hline 9.1523 - 10 \end{array}$$

or:

$$9.1523 - 10 = \overline{1}.1523$$

EXAMPLE:

Add the logarithms 3.7076 and 6.6893 - 10.

$$\begin{array}{r} 3.7076 \\ 6.6893 - 10 \\ \hline 10.3969 - 10 \end{array}$$

or:

$$10.3969 - 10 = 0.3969$$

EXAMPLE:

Subtract the logarithm 4.6721 from the logarithm 3.7076.

To subtract a larger logarithm from a smaller, add 10 or a multiple of 10 to the smaller logarithm, and subtract the same number from the logarithm by writing the number added with a minus sign to the right of the logarithm.

$$\begin{array}{r} 3.7076 = 13.7076 - 10 \\ 4.6721 \\ \hline 9.0355 - 10 \end{array}$$

or:

$$9.0355 - 10 = \overline{1}.0355$$

EXAMPLE:

Subtract the logarithm 3.4298 from the logarithm 1.5224.

$$\begin{array}{r} 1.5224 = 11.5224 - 10 \\ 3.4298 \\ \hline 8.0926 - 10 \end{array}$$

or:

$$8.0926 - 10 = \overline{2}.0926$$

EXERCISE 31:

Perform the indicated operations.

1. $(\bar{3}.6964)+(4.0775-10)$ 2. $(\bar{4}.7067)+(\bar{2}.7067)$ 3. $(2.7236)+(3.4036)$

4. $(4.7493)+(3.4289)$ 5. $(2.4287)+(\bar{5}.3982)$ 6. $(3.1289)-(2.7263)$

7. $(1.4298)-(\bar{1}.6618)$ 8. $(5.5510)-(4.6721)$ 9. $(\bar{3}.5224)-(3.6964)$

10. $(6.4771)-(2.8149)$

59-29. Multiplication Using Logarithms

LAW: The logarithm of a product is equal to the sum of the logarithms of the factors.

Consider the two factors 10 and 100. The common logarithm of the two factors are 1 and 2 respectively.

Therefore: $1 = \log\ 10$ (1)

and: $2 = \log\ 100$ (2)

Writing equation (1) in exponential form:

$$10^1 = 10$$

Writing equation (2) in exponential form:

$$10^2 = 100$$

Therefore: $10 \times 100 = 10^1 \times 10^2$

When multiplying two factors with exponents, their exponents are added. Keep in mind that an exponent is a logarithm.

Therefore: $10^1 \times 10^2 = 10^{1+2} = 10^3$

$$\log\ (10 \times 100) = 1 + 2 = \log\ 10 + \log\ 100$$

The latter equation stated as a law is:

Multiplication factors may be accomplished by adding their logarithms, or LOG A x B = LOG A + LOG B.

EXAMPLE:

Find the product of 2.1 x 336.

Let X equal the desired product.

Then:

$$X = 2.1 \times 336$$

Taking the logarithm of both sides:

$$\log\ X = \log\ (2.1 \times 336)$$

Applying the law of logarithms log AB = log A + log B:

$$\log X = \log 2.1 + \log 336$$

Determining the logs from the tables and adding:

$$
\begin{array}{rl}
\log 2.1 = & 0.3222 \\
\log 336 = & \underline{2.5263} \\
& 2.8485
\end{array}
$$

Therefore:

$$\log X = 2.8485$$

To solve the equation for X, take the antilog of both members.

$$\log X = 2.8485$$

$$\text{antilog } \log X = \text{antilog } 2.8485$$

The antilog of log X equals X.

Therefore:

$$X = \text{antilog } 2.8485$$

$$X = 7.06 \times 10^2$$

$$X = 706$$

EXAMPLE:

Given E = IR. Find the value of E when I equals 0.000326 and R equals 621,000.

$$\log E = \log IR$$

$$\log E = \log I + \log R$$

$$\log E = \log 0.000326 + \log 621,000$$

$$\log E = \overline{4}.5132 + 5.7931$$

$$
\begin{array}{r}
6.5132 - 10 \\
\underline{5.7931} \\
12.3063 - 10
\end{array}
$$

$$\log E = 2.3063$$

$$\text{antilog } E = \text{antilog } 2.3063$$

$$E = 2.02 \times 10^2$$

$$E = 202$$

EXERCISE 32:

Find the product of the following through the use of logarithms.

1. 0.7883 x 125   2. 5 x 3.142 x 528

3. 31.62 x 4.73   4. 37.9 x 4.08 x 0.864

59-30.   Division Using Logarithms

The logarithm of the quotient of two numbers is the logarithm of the dividend minus the logarithm of the divisor.   As with multiplication, this rule is simply an application of the laws of exponents.

Consider the two factors 10 and 100.   The common logarithms of the numbers are 1 and 2 respectively.

Therefore: $1 = \log 10$ (1)

                                                  $2 = \log 100$ (2)

Writing equation (1) in exponential form:

$$10^1 = 10$$

Writing equation (2) in exponential form:

$$10^2 = 100$$

Therefore: $\dfrac{100}{10} = \dfrac{10^2}{10^1} = 10^{2-1} = 10^1$

Since the exponents are logarithms:

$$\log \frac{100}{10} = 2 - 1 = \log 100 - \log 10$$

Stating the latter equation as a general law:

Division of factors may be accomplished by subtracting their logarithms or $\text{LOG} \dfrac{A}{B} = \text{LOG A} - \text{LOG B}$.

EXAMPLE:

Find the quotient of 37. 4/ 1. 7 by the use of logarithms.

Let X equal the desired quotient.

Therefore: $X = 37.4/1.7$

Stated in terms of logarithms:

$$\log X = \log 37.4 - \log 1.7$$

Finding the logarithms of the numbers and subtracting:

$$
\begin{array}{rl}
& \log 37.4 = 1.5729 \\
(\text{minus}) & \log 1.7 \ = 0.2304 \\
\hline
& 1.3425
\end{array}
$$

Therefore: $\log X = 1.3425$

Solving the equation by taking the antilog of both members of the equation:

$$\text{antilog} \log X = \text{antilog } 1.3425$$

$$X = \text{antilog } 1.3425$$

$$X = 2.20 \times 10^1$$

$$X = 22$$

or: $$\frac{37.4}{1.7} = 22$$

EXAMPLE:

Find the quotient of $\frac{16.3}{0.008}$

Let X equal the desired quotient:

Therefore: $$X = \frac{16.3}{0.008}$$

Taking the log of both members of the equation:

$$\log X = \log \frac{16.3}{0.008}$$

Applying the law of logarithms:

$$\log X = \log 16.3 - \log 0.008$$

Finding the logarithms and subtracting:

$$\begin{array}{l} \log 16.3 = 11.2122 - 10 \\ \log 0.008 = \underline{-7.9031 - 10} \\ 3.3091 \end{array}$$

Notice that the characteristic in the minuend $\overline{7}$ was changed from the characteristic -3.

Therefore: $\log X = 3.3091$

Taking the antilog of both members:

$$\text{antilog } \log X = \text{antilog } 3.3091$$

$$X = \text{antilog } 3.3091$$

$$X = 2.04 \times 10^3$$

$$X = 2,040$$

or: $$\frac{16.3}{0.008} = 2,040$$

EXERCISE 33:

Find the quotient of the following by use of logarithms.

1.   635.6/25.4   2.   0.26/0.061   3.   0.126/0.00543

4.   874/26.3   5.   632/0.102

59-31. Raising a Number to a Power by the Use of Logarithms

LAW: The logarithm of a power of a number equals the logarithm of the number multiplied by the exponent of the number.

$$\text{LOG } B^A = A \text{ LOG } B$$

EXAMPLE:

Find the value of $(18.5)^5$.

Let X equal the desired result.

Then: $X = (18.5)^5$

Taking the logarithm of both members:

$$\log X = \log (18.5)^5$$

Applying the law of logarithms:

$$\log X = 5 \log (18.5)$$

$$\log 18.5 = 1.2672$$

$$\log X = 6.3360$$

$$\text{antilog } \log X = \text{antilog } 6.3360$$

$$X = \text{antilog } 6.3360$$

$$\text{antilog } 6.3360 = 2.17 \times 10^6$$

$$X = 2,170,000$$

or: $(18.5)^5 = 2,170,000$

59-32.   Extracting a Root by the Use of Logarithms

LAW: The logarithm of the root of a number is equal to the logarithm of the number divided by the root.

EXAMPLE:

Evaluate $\sqrt[5]{327}$

Let X equal the desired result.

Therefore:

$$X = \sqrt[5]{327}$$

$$\log X = \log \sqrt[5]{327}$$

$$\log X = \log (327)^{1/5}$$

$$\log X = \frac{1}{5} \log 327$$

$$\log 327 = 2.5145$$

Therefore:

$$\log X = \frac{1}{5} (2.5145)$$

$$\log X = 0.5029$$

$$\text{antilog log X} = \text{antilog } 0.5029$$

$$X = \text{antilog } 0.5029$$

$$\text{antilog } 0.5029 = 3.18$$

$$X = 3.18$$

or:

$$\sqrt[5]{327} = 3.18$$

TRIGONOMETRY

TRIGONOMETRY is the science of measuring the sides and angles of triangles. In the study of trigonometry, use is made of the fact that a definite relationship exists between angles and their sides. These relationships (called trig functions) have been named and defined. They form the nucleus of trigonometry.

59-33. Definitions
Before trigonometry can be applied to problem solving, some basic definitions must be given.

Angles: An ANGLE is formed when two lines meet at a point. The two lines are called the SIDES of the angle, and the meeting point of the lines is called the VERTEX of the angle. Figure 59-2 shows a graphical representation of an angle.
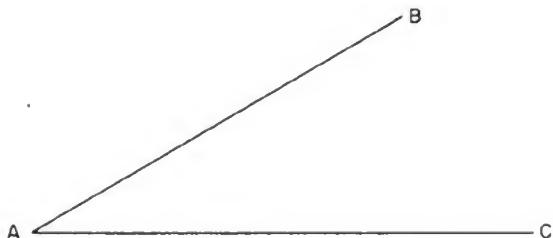
Figure 59-2 - Acute angle.

The symbol used to represent an angle is ∠ . Frequently, letters of the Greek alphabet are used to represent angles. One of the most widely used Greek symbols is θ, pronounced THETA.

Angles can be generated by a revolving line. If the line AB in Figure 59-3 is rotated about a point (A), an angle is formed. The magnitude of the angle is given in reference to the STARTING or INITIAL POINT. The dotted line, AC, in Figure 59-3, which could have been a solid line, is called the LEADING SIDE. The line AB is the TERMINAL SIDE.
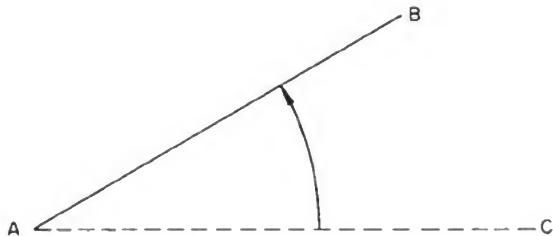
Figure 59-3 - Acute angle.

The magnitude of an angle is generally expressed in DEGREES. If the terminal side of an angle is rotated through a full revolution, it is said to have generated an angle of three hundred and sixty degrees. Numerically, this angle can be represented as 360°. Of course, an angle greater or smaller than 360° can be generated.

Since a line having gone through a complete revolution has also gone through 360°, one degree may be defined as the angle generated when a line has rotated 1/360 of a full revolution. The degree is further divided into MINUTES and SECONDS. One sixtieth of a degree is a minute. One sixtieth of a minute is a second.

EXERCISE 34:

1. How many degrees are there in 1/4 of a revolution, 1/2 revolution, and 3/4 revolution?

2. How many degrees are there in two revolutions, three revolutions, and eight revolutions?

Acute Angle: An ACUTE ANGLE is an angle less than ninety degrees.

Obtuse Angle: An OBTUSE ANGLE is an angle greater than ninety degrees.

Right Angle: A RIGHT ANGLE is equal to ninety degrees.

Negative Angle: A NEGATIVE ANGLE is one which is generated with clockwise rotation of the terminal sides.

Complimentary Angles: COMPLIMENTARY ANGLES are two angles the sum of which is equal to ninety degrees.

Supplementary Angles: SUPPLEMENTARY ANGLES are two angles the sum of which is equal to one hundred and eighty degrees.

Triangles: A TRIANGLE is a geometrical figure having three sides (sometimes called legs) and three angles. The sum of the angles of a triangle is equal to 180°. Figure 59-4 shows four types of triangles.

The ISOSCELES triangle has two equal sides, the EQUILATERAL triangle has three equal sides, and the SCALENE triangle has no equal sides.

The RIGHT triangle is considered here as a special case because it is important to the study of basic trigonometry.
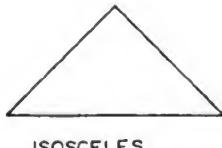
ISOSCELES
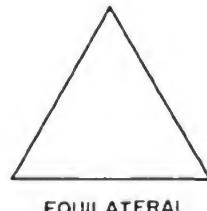
EQUILATERAL
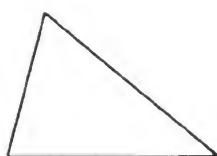
SCALENE

RIGHT

Figure 59-4 - Types of triangles.

Right Triangle: A right triangle is a triangle which has one ninety degree angle. The trigonometric functions are defined using the right triangle. Figure 59-5 shows a right triangle with its sides and angles labeled.
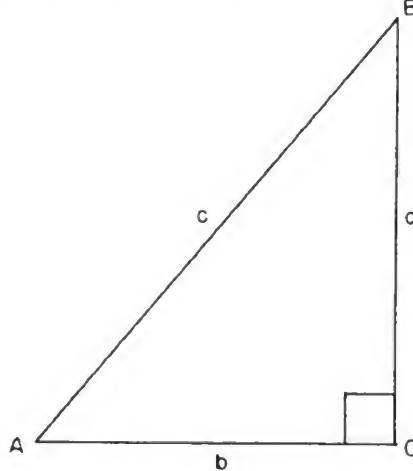
Figure 59-5 - Right triangle.

Side c is called the HYPOTENUSE of the right triangle. The hypotenuse is the longest side of the triangle, and it is opposite the largest angle (90° angle).

The relationship which exists between the sides of a right triangle is described by the PYTHAG-OREAN THEOREM.

Pythagorean Theorem: This theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides. This relationship may be expressed mathematically as:

$$c^2 = a^2 + b^2$$

To solve for the hypotenuse, the square root of both sides of the equation is extracted.

This gives: $c = \pm\sqrt{a^2 + b^2}$

A negative square root has no meaning here. Therefore, the final form is:

$$c = \sqrt{a^2 + b^2}$$

To solve for side b, subtract a^2 from each member of the equation.

This gives: $b^2 = c^2 - a^2$

Since the first power of b is desired, extract the square root of both sides of the equation.

Therefore:

$$b = \sqrt{c^2 - a^2}$$

The negative root is ignored because it has no meaning in this application.

Side a may be solved in the same manner.

$$a = \sqrt{c^2 - b^2}$$

A graphical representation of the theorem is frequently shown in the following manner:

—
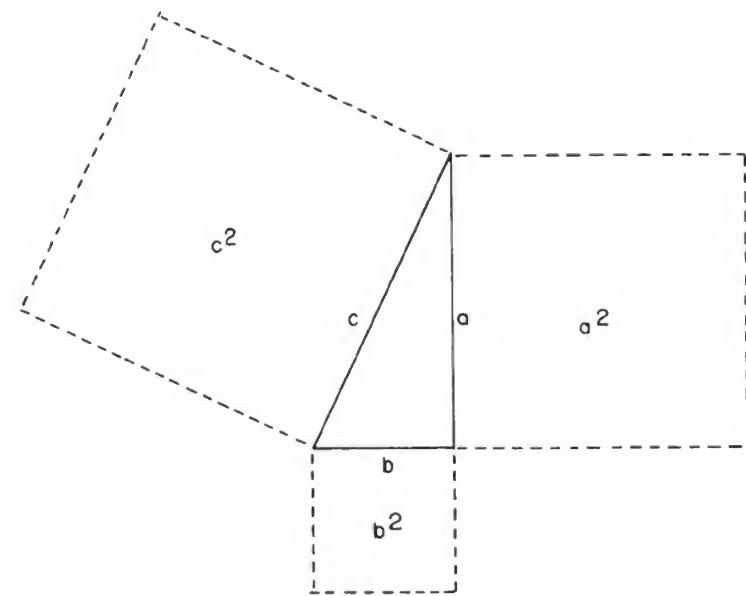
Figure 59-6 - Pythagorean theorem.

The square on the hypotenuse is equal to the sum of the squares on the legs.

EXERCISE 35:

1. One leg of a triangle is twice as long as the other. If the hypotenuse is 10 units long, how long are the two legs?

2. What is the length of the hypotenuse in the following diagram?
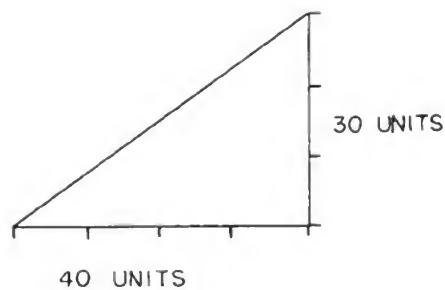
Figure 59-7 - Application of Pythagorean theorem.

3. A triangle is to be constructed with a hypotenuse of 100 units and a vertical leg of 50 units. How long will be the horizontal leg?

59-34. Trigonometric Ratios of Acute Angles

Figure 59-8A shows a right triangle with the angles labeled θ (THETA), ϕ (PHI), and α (ALPHA).

Alpha is the right angle of the right triangle ABC. If the three sides are used two at a time, the following ratios of angle θ may be expressed as:

$$\frac{a}{c}, \ \frac{b}{c}, \ \frac{a}{b}$$
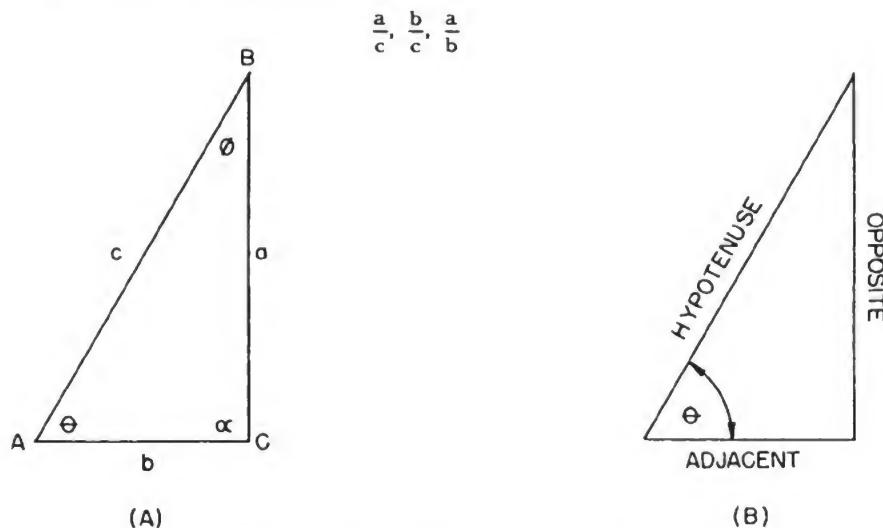
Figure 59-8 - Triangles (right).

For identification purposes, they have been assigned the following names. The ratio a/c is identified as the SINE ratio of the angle θ. This is normally written sin θ. The ratio b/c is identified as the COSINE ratio of the angle θ. It is normally written as cos θ.

The ratio a/b is identified as the TANGENT ratio of the angle θ. It is normally written as tan θ.

These trigonometric ratios are easy to remember if the position of two sides of the triangle are considered with respect to the acute angle under consideration. In Figure 59-8B, side a is opposite to the angle θ. Thus, side a is called the opposite side in respect to the angle theta. Side b is called the adjacent side in respect to the angle theta. The trigonometric ratios can be expressed in terms of these sides in the following manner:

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$

These trigonometric ratios should be memorized so that the ratio of either acute angle of a right triangle may be found, regardless of its position.

The numerical value of the trigonometric functions depends only on the magnitude of the angle. This is shown in Figure 59-9. The angle θ is generated by revolving the line AC about the point A. From the points D and B, perpendiculars are dropped to the initial line or adjacent side AZ. This process will form SIMILAR RIGHT TRIANGLES ADE and ABF because each of the triangles are right angles having the common angle θ.

Hence:

$$\sin \theta = \frac{DE}{AD} = \frac{FB}{AB}$$
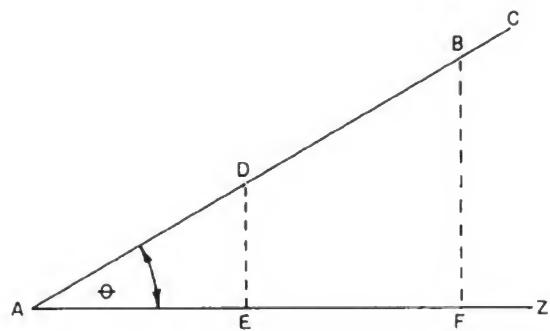
Figure 59-9 - Similar triangles.

$$\cos \theta = \frac{AE}{AD} = \frac{AF}{AB}$$

$$\tan \theta = \frac{DE}{AE} = \frac{BF}{AF}$$

The sin θ of the triangle ADE is equal to the sin θ of the larger triangle AFB. Therefore, it should be evident that the size of the right triangle is immaterial and that only the ratios of the sides are important.

Each of the ratios will change in value whenever the angle is increased or decreased. Therefore, the value of the trigonometric ratios are really functions of the angle considered.

EXAMPLE:

Determine the sin, cos and tan functions of the acute angles θ and $\emptyset$ in the right triangle shown in Figure 59-10 if R = 4 in, X = 3 in, and Z = 5 in.
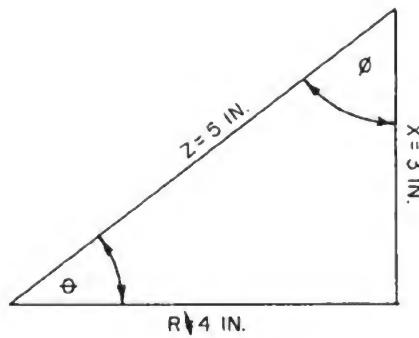
Figure 59-10 - Determining functions.

Applying the definitions of the sin, cos, and tan.

$$\sin \theta = \frac{X}{Z} = \frac{3 \text{ in}}{5 \text{ in}} = \frac{3}{5} = 0.6$$

$$\sin \emptyset = \frac{R}{Z} = \frac{4 \text{ in}}{5 \text{ in}} = \frac{4}{5} = 0.8$$

$$\cos \theta = \frac{R}{Z} = \frac{4 \text{ in}}{5 \text{ in}} = \frac{4}{5} = 0.8$$

$$\cos \emptyset = \frac{X}{Z} = \frac{3 \text{ in}}{5 \text{ in}} = \frac{3}{5} = 0.6$$

$$\tan \theta = \frac{X}{R} = \frac{3 \text{ in}}{4 \text{ in}} = \frac{3}{4} = 0.75$$

$$\tan \emptyset = \frac{R}{X} = \frac{4 \text{ in}}{3 \text{ in}} = \frac{4}{3} = 1.333$$

59-35. Constructing an Acute Angle When One Trigonometric Function is Given
    When given the trigonometric function of an acute angle, the angle may be constructed geometrically from the definitions of the given function.

EXAMPLE:

    Construct the acute angle A of a right triangle if the tangent of the angle equals 0.5.

Step One:

    Construct two perpendicular lined AC and BC as shown in Figure 59-11.

Figure 59-11 - Constructing an acute angle.

Step Two:

    Measure off, with a divider, 1 unit along line BC and label that point B'. Measure off 2 units along line AC and label that point A' as shown in Figure 59-12.

Step Three:

    Join the points A' and B' forming the right triangle A'B'C as shown in Figure 59-13.
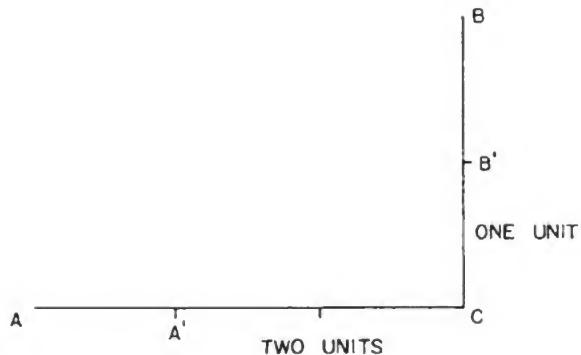
Figure 59-12 - Locating points on the lines of an acute angle.
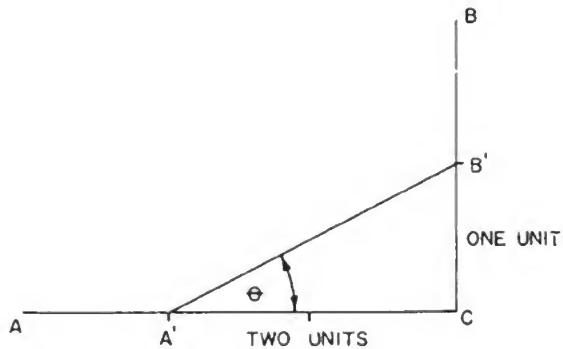
Figure 59-13 - Forming a right triangle.

Step Four:

Tan θ equals 1/2. Therefore, angle θ is the required angle.

59-36. Trigonometric Functions of any Angle

In the preceding section, the discussion of angles was expanded to include both positive and negative angles in any QUADRANT. Angles of more than 90° are normally illustrated in a system of RECTANGULAR COORDINATES. This system is composed of an X and Y axis as shown in Figure 59-14. The distance from the origin to a point on the X axis is called the ABSICCA. The distance from the origin to a point on the Y axis is called the ORDINATE. The individual squares formed when the lines are mutually perpendicular to one another are the quadrants. The quadrants are usually designated using the first four Roman numerals. The quadrant in the upper right-hand corner is designated as the first quadrant (I). The upper left-hand quadrant is known as the second quadrant (II). The lower left-hand quadrant is the third (III) and the lower right-hand quadrant is the fourth (IV).

Since the size of an angle in a right triangle must be less than 90°, the functions have been restricted to acute angles. However, all angles have sines, cosines and tangents.

Consider the line in Figure 59-15 which represents a radius vector designated as Z. The radius vector is revolving around a point 0 in the system in a counterclockwise direction. The line is
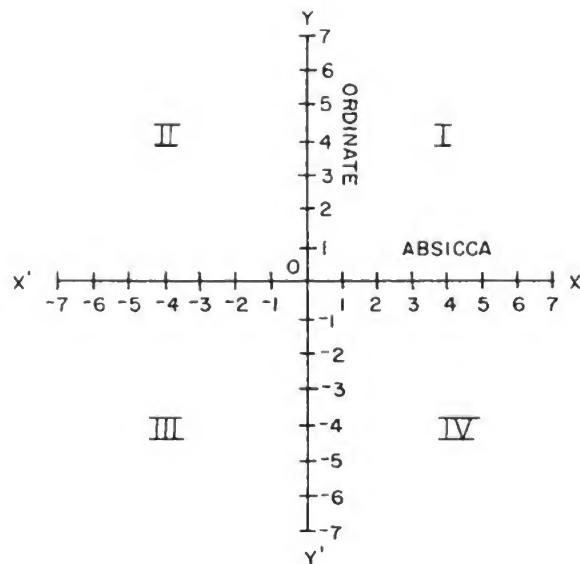
Figure 59-14 - System of rectangular coordinates.
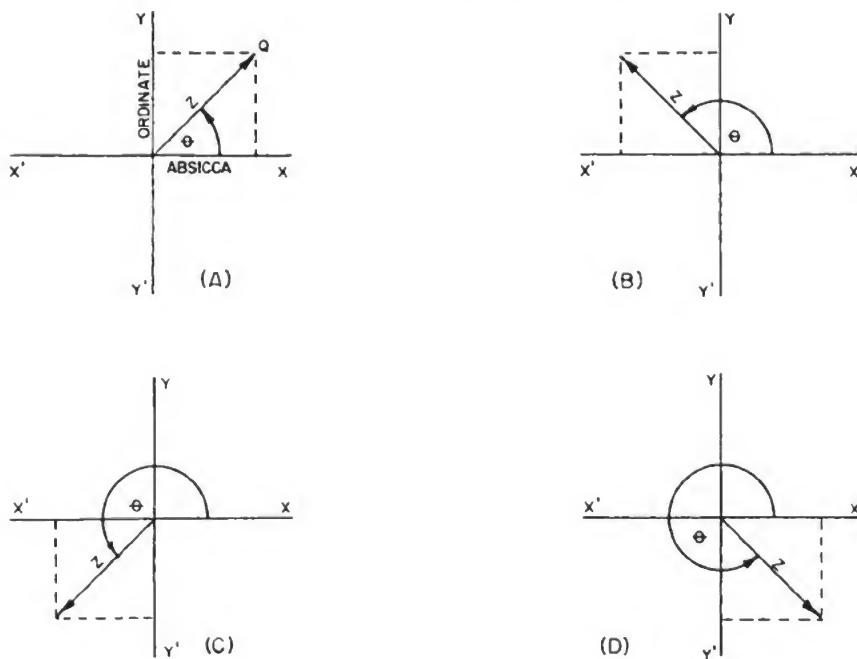
Figure 59-15 - Angles in various quadrants.

generating an angle which is labeled angle θ. The initial side corresponds to the positive X axis, and the terminal side corresponds to the line Z. If a perpendicular is dropped from any point along the line, Z, in any quadrant, a right triangle is formed (XYZ).

The hypotenuse of the right triangle will be a constant length equal to Z with the legs (X and Y) having

lengths equal to the projection of the radius vector on the X and Y axes respectively. Thus, the trigonometric ratios of angles in a rectangular coordinate system is defined as:

$$\sin \theta = \frac{\text{ordinate}}{\text{radius}} = \frac{Y}{Z}$$

$$\cos \theta = \frac{\text{absicca}}{\text{radius}} = \frac{X}{Z}$$

$$\tan \theta = \frac{\text{ordinate}}{\text{absicca}} = \frac{Y}{X}$$

The value of the trigonometric functions depends only upon the size of the angle. Therefore, for every angle, there is only one value for each function.

59-37. Signs of Functions

The signs of the functions of angles in various quadrants are determined by the signs of the X and Y coordinates. Figure 59-14, the rectangular coordinate system, showed the signs on the X and Y axes. The X axis is positive to the right of the origin, and negative to the left of the origin. The Y axis is positive above the origin, and the negative below the origin. The quadrants will also have a positive and negative sign associated with them for each trigonometric function.

EXAMPLE:

The signs of the trigonometric functions in each quadrant are:

First quadrant: All of the functions are positive.

Second quadrant: Only the sine is positive.

Third quadrant: Only the tangent function is positive.

Fourth quadrant: Only the cosine function is positive.

Quadrant	$\sin \theta$	$\cos \theta$	$\tan \theta$
I	+	+	+
II	+	-	-
III	-	-	+
IV	-	+	-

TABLE 59-3. Signs of Functions

59-38. Table of Functions

Trigonometric tables are lists of the numerical values of the ratios of sides of right triangles for angles from 0° to 90°. If it is desired to know the ratio of two sides of a right triangle containing a known acute angle θ, look for the angle θ in the trig table and find the desired ratio. This table is provided at the end of the volume.

The proper ratio may be found for the various angles from 0° to 90° by observing the following procedure.

EXAMPLE:

Find the angle whose cosine is 0.7059, in the trig tables.

Expressing in notation: θ = arc cos 0.7059

Expression is read: Theta equals an arc whose cosine function is 0.7059.

Step One:

Find the numerical value 0.7059 in the trig tables in the row marked cos.

Step Two:

Reading 0.1 at the top of the column and the number 45 at the left of the row, the angle is equal to 45.1°.

EXAMPLE:

Find the angle whose tangent is equal to 1.

Expressing in notation: θ = arc tan 1

Step One:

Find the number (1) in the tangent row of the tables.

Step Two:

Reading 0.0° at the top of the column, and 45° at the left of the row, the angle is equal to 45°.

59-39.   Reducing the Function of Any Angle to the Function of an Acute Angle

To find the trigonometric function of any angle θ, find the same function of the acute angle formed by the terminal side and the horizontal axis and prefix the proper algebraic sign for that function and quadrant.

When finding functions of angles greater than 90°, make a diagram showing the approximate location of the angle.   This will avoid the possibility of error.

EXAMPLE:

Find the sin of 210°.

Step One:

Make a diagram rotating the radius vector 210° from the positive X axis as shown in Figure 59-16.
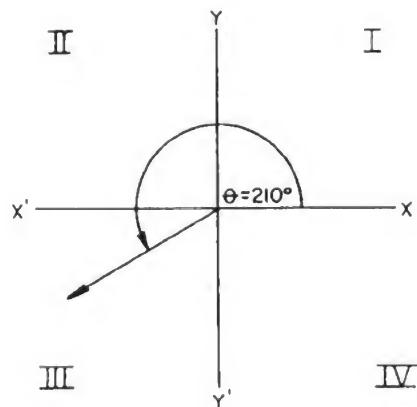
Figure 59-16 - Determining the function of an angle greater than 90°.

Step Two:

Determine the algebraic sign of the sine function in quadrant III **as** shown in Figure 59-17.

$$\sin \theta = \frac{y''}{z} = \frac{-}{+} = -$$
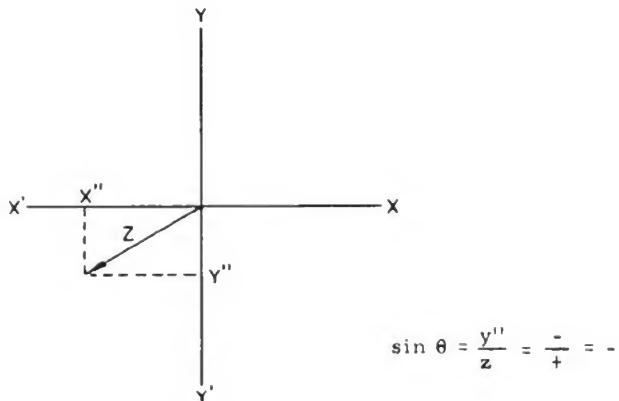
Figure 59-17 - Locating the sign of an angle greater than 90°.

Step Three:

Determine the number of degrees between the radius vector and the horizontal axis. This operation, shown in Figure 59-18, will give the angle which is equivalent to 210°.

$$210° - 180° = 30°$$
$$\text{or}$$
$$\sin 210° = - \sin 30°$$
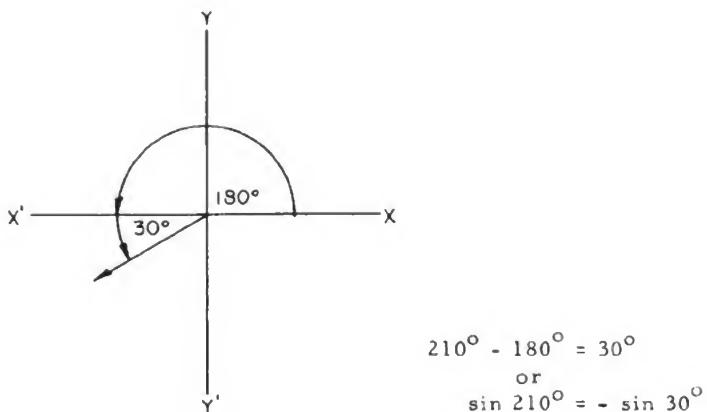
Figure 59-18 - Locating equivalent angle.

Step Four:

Find the sin of 30° in the trig tables.

$$\sin 210° = - \sin 30°$$

$$\sin 30° = 0.5$$

therefore: $$\sin 210° = (-1)(0.5)$$

$$= -0.5$$

EXAMPLE:

Find the cosine of 855°.

Step One:

Make a diagram rotating the radius vector 855° from the positive axis in the counterclockwise direction as shown in Figure 59-19.
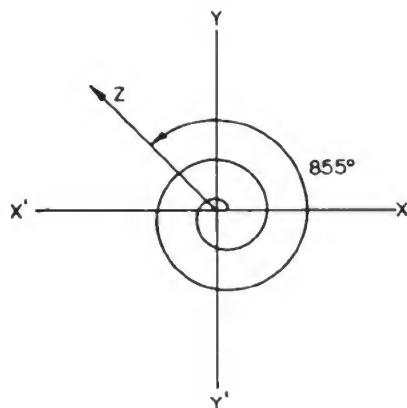
Figure 59-19 - Finding the cos of 855°.

Step Two:

Determine the algebraic sign of the cos function in quadrant II as shown in Figure 59-20.

$$\cos \theta = \frac{X}{Z} = \frac{-}{+} = -$$
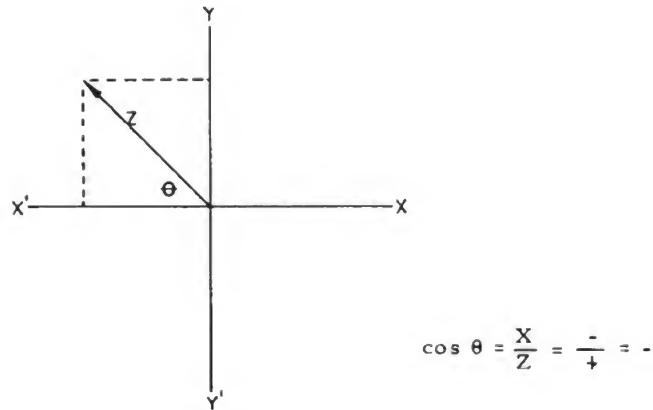
Figure 59-20 - Determining the algebraic sign.

Step Three:

Determine the number of degrees between the radius vector and the horizontal axis which will give the angle equivalent to 855° as shown in Figure 59-21.

Step Four:

Find the cos of 45°.

$$\cos 855° = - \cos 45°$$

$$180^{\circ} - 135^{\circ} = 45^{\circ}$$
$$\text{or}$$
$$\cos 855^{\circ} = - \cos 45^{\circ}$$
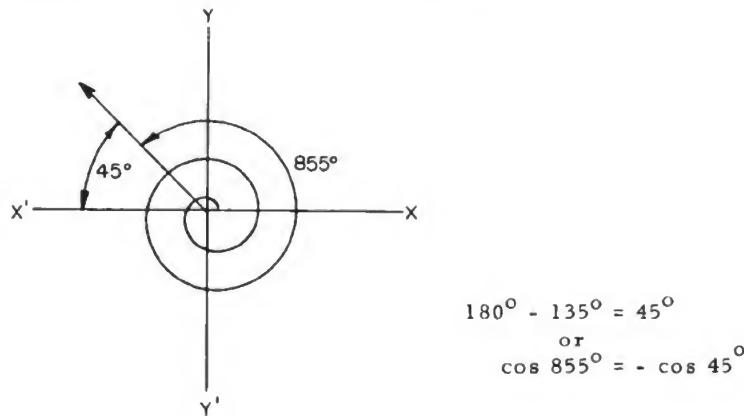
Figure 59-21 - Determining the equivalent angle.

$$\cos 45^{\circ} = 0.707$$

therefore: $$\cos 855^{\circ} = (-1)(0.707)$$

$$= -0.707$$

EXERCISE 36:

Solve the following problems:

1. $\sin 210^{\circ}$ 2. $\cos 60^{\circ}$ 3. $\tan 310.4^{\circ}$ 4. $\sin 608^{\circ}$

5. $\cos 167.8^{\circ}$ 6. $\cos 720.4^{\circ}$ 7. $\tan 68.9^{\circ}$ 8. $\tan 341.6^{\circ}$

9. $\sin 135.4^{\circ}$ 10. $\cos 99.8^{\circ}$

59-40. Solving a Right Triangle when an Acute Angle and the Hypotenuse are Given:

EXAMPLE:

Find the unknown side R and X, and the value of angle Phi (ϕ) in the right triangle ABC in Figure 59-22 if angle theta (θ) is 30° and the hypotenuse, Z, is 50.
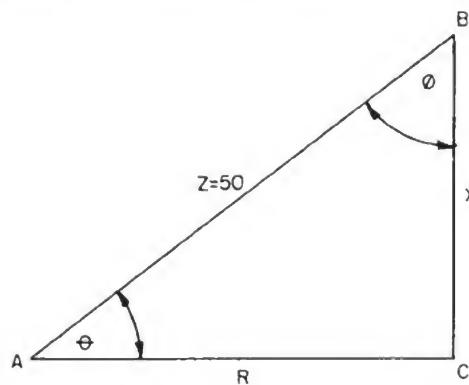
Figure 59-22

$$\sin \theta = X/Z$$

Transposing for X: $X = Z \sin \theta$

Substituting the value of Z and $\sin \theta$:

$$X = (50)(0.5) = 25$$

Solving for R using the trigonometric ratio $\cos \theta$ because it contains the two known elements $\cos \theta$ and Z, and the unknown element R:

$$\cos \theta = R/Z$$

Transposing for R: $R = Z \cos \theta$

Substituting the value Z and $\cos \theta$:

$$R = (50)(0.866) = 43.3$$

The relationship between the two acute angles of a right triangle is:

$$\theta = 90^{\circ} - \emptyset$$

Transposing for $\emptyset$: $\emptyset = 90^{\circ} - \theta$

Substituting the value of θ: $\emptyset = 90^{\circ} - 30^{\circ}$

$$\emptyset = 60^{\circ}$$

Therefore: $\emptyset = 60^{\circ}$

$$R = 43.3$$

$$X = 25$$

This solution can be checked by using the Pythagorean theorem which states that $Z^2 = X^2 + R^2$. Thus, substituting values in

$$Z^2 = X^2 + R^2$$

$$(50)^2 = (25)^2 + (43.3)^2$$

$$2500 = 2500$$

Since the trigonometric functions are rounded off, the check shows the solution to be correct for this degree of accuracy.

EXAMPLE:

Find the unknown sides R and X, and the value of $\emptyset$ in the right triangle ABC in Figure 29-23 if angle $\theta = 45^{\circ}$ and the hypotenuse Z is 60.

Solve for the value of X using the trigonometric ratio $\sin \theta$ because it contains the two known elements $\sin \theta$ and Z, and the unknown element X.

$$\sin \theta = X/Z$$

Transposing for X: $X = Z \sin \theta$
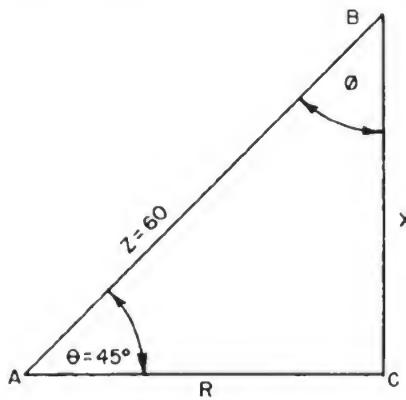
Figure 59-23 - Hypotenuse and theta given.

Substituting the values of sin θ and Z:

$$X = (60) \sin 45^\circ$$

$$X = (60)(0.707)$$

$$X = 42.42$$

Solve for the value R using the trigonometric ratio cos θ because it contains the two known elements cos θ and Z, and the unknown element R.

$$\cos \theta = R/Z$$

Transposing for R: $$R = Z \cos \theta$$

Substituting values of θ and Z:

$$R = 60 \cos 45^\circ$$

$$R = 60 (0.707)$$

$$R = 42.42$$

The relationship between θ and ∅ is $\theta + \emptyset = 90^\circ$.

Transposing for ∅: $$\emptyset = 90^\circ - \theta$$

Substituting 45° for θ: $$\emptyset = 90^\circ - 45^\circ$$

$$\emptyset = 45^\circ$$

Check by use of the Pythagorean theorem:

$$Z^2 = X^2 + R^2$$

Substituting values:

$$(60)^2 = (42.42)^2 + (42.42)^2$$

$$3600 = 3600$$

59-41. Solving a Right Triangle When Given an Acute Angle and a Side not the Hypotenuse:

EXAMPLE:

   Given: X = 50 and θ = 60°, solve for Z and R.
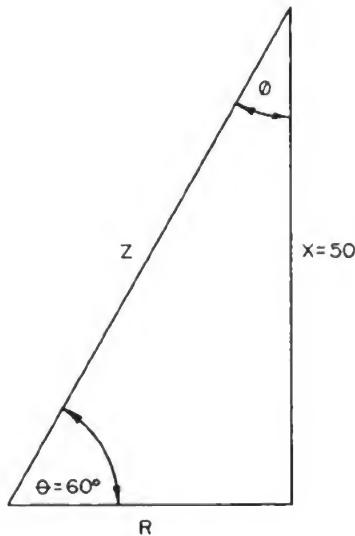
Figure 59-24 - Opposite side and theta given.

Solve for Z using the trigonometric ratio sin θ because it contains the two known elements sin θ and X, and the unknown element Z.

$$\sin \theta = X/Z$$

Solving for Z: $X = Z \sin \theta$

Transpose: $Z = X/\sin \theta$

Substituting the value of X and sin θ:

$$Z = 50/\sin 60°$$

$$Z = 50/0.866$$

$$Z = 57.7$$

Solve for the value of R using the trigonometric ratio tan θ because it contains the two known elements tan θ and X, and the unknown element R.

$$\tan \theta = X/R$$

Solving for R: $X = R \tan \theta$

Transposing: $R = X/\tan \theta$

Substituting the value for X and tan θ:

$$R = 50/\tan 60^\circ$$

$$R = 50/1.7321$$

$$R = 28.9$$

Check by use of the Pythagorean theorem:

$$Z^2 = X^2 + R^2$$

Substituting the values:

$$(57.7)^2 = (50)^2 + (28.9)^2$$

$$3329.29 = 3329.29$$

59-42. Solving a Right Triangle When Given Two Sides Other Than the Hypotenuse

EXAMPLE:

Given: X = 40 and R = 30. Shown in Figure 59-25. Find Z and angle θ.
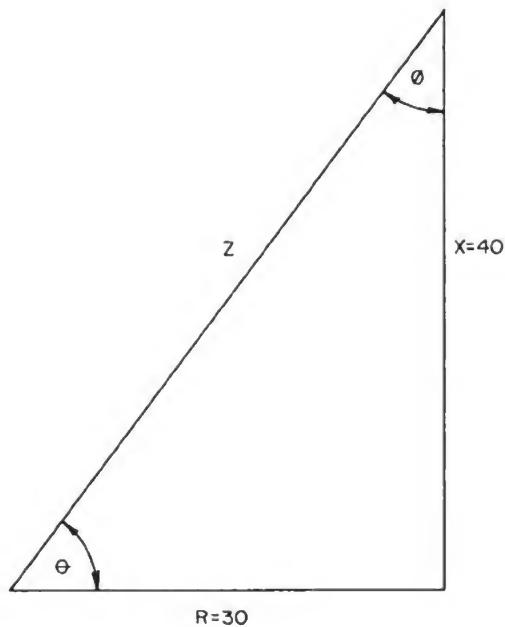
Figure 59-25 - Opposite and adjacent side given.

In this type of problem always solve for the angle theta (θ) first.

Use the trigonometric function tan θ because it contains the two known elements X and R, and the unknown element θ.

$$\tan \theta = X/R$$

Solving the equation for θ:

$$\theta = arc \ tan \ X/R$$

Expression is read: θ equals the angle whose tan function is X/R.

Substituting the values of X and R:

$$\theta = arc \ tan \ 40/30$$

$$\theta = arc \ tan \ 1.33$$

$$\theta = 53.1^{o}$$

When solving for Z, use either the sin or cos ratio of θ because each will give an equation containing two known elements and one unknown element.

$$\sin \theta = X/Z$$

Solving for Z: $Z \sin \theta = X$

Transposing: $Z = X/\sin \theta$

Substituting the values of X and sin θ:

$$Z = 40/\sin \ 53.1^{o}$$

$$Z = 40/0.7997$$

$$Z = 50$$

Check by use of Pythagorean theorem:

$$Z^2 = X^2 + R^2$$

$$(50)^2 = (40)^2 + (30)^2$$

$$2500 = 2500$$

VECTORS

Many common physical quantities such as temperature, the speed of a moving object, or the displacement of a ship can be expressed as a certain number of units. These units define only the magnitude and give no indications of the direction in which the quantity acts. Such quantities are called SCALAR quantities. If both the magnitude and the direction in which the quantity acts are indicated, it is a VECTOR quantity. A vector representing the speed and heading of a ship having a speed of 10 knots and a heading of 45° (northeast) is a straight line extending upward and to the right. The length of the line is proportional to the speed of 10 knots. The angle that the line makes with the vertical (north at the top) is 45° clockwise from the vertical.

59-43. Operator J
It is often necessary to perform operations involving the square root of a negative number, for example, -9, -5, and $-x$. Because no real number when multiplied by itself will produce a negative result, the roots of numbers such as the foregoing cannot be extracted in the real number system. It therefore becomes necessary to introduce a new type of notation to indicate the square root of a negative number. These numbers are called IMAGINARY NUMBERS to distinguish them from the REAL NUMBERS. Actually, the numbers that are called imaginary in the mathematical sense are real in the physical sense. The term is merely one of convenience, as will be pointed out in the succeeding paragraphs.

In algebra, the foregoing quantities are treated as $\sqrt{-1}$, $\sqrt{9}$, or $\sqrt{-1} \times 3$; $\sqrt{-1}\sqrt{5}$; and $\sqrt{-1}\sqrt{x}$. The term, $\sqrt{-1}$, is expressed as i (for imaginary) in mathematics books, but when working with electrical circuits it is convenient to use the term j (called the J OPERATOR), because i is used to indicate the instantaneous value of the circuit current.

In order to present a quantity graphically, some system of coordinates must be employed. Quantities involving the j operator may be conveniently expressed by the use of RECTANGULAR COOR-DINATES, as shown in Figure 59-26. In order to specify a vector in terms of its X and Y components, some means must be employed to distinguish between X-axis and Y-axis projections. Be-cause the +Y axis projection is +90° from the +X axis projection, a convenient operator is one that will, when applied to a vector, rotate it without altering the magnitude of the vector. Let +j be such an operator that produces 90° COUNTERCLOCKWISE rotation of any vector to which it is applied as a multiplying factor. Also, let -j be such an operator that produces 90° CLOCKWISE rotation of any vector to which it is applied as a multiplying factor.

Successive application of the operator +j to a vector will produce successive 90° steps rotation of the vector in the counterclockwise direction without affecting the magnitude of the vector. Likewise, successive applications of the operator -j will produce successive 90° steps of rotation in the clock-wise direction. This rotation is shown in Table 59-4.

Operator	Mathematical Equivalent	Direction of Rotation	Degree of Rotation
j	$\sqrt{-1}$	ccw	90
j^2	-1	ccw	180
j^3	$-\sqrt{-1}$	ccw	270
j^4	1	ccw	360
-j	$-\sqrt{-1}$	cw	-90
$(-j)^2$	-1	cw	-180
$(-j)^3$	$\sqrt{-1}$	cw	-270
$(-j)^4$	1	cw	-360

ccw - counterclockwise
cw - clockwise

TABLE 59-4.   Relation of Operation J to Vector Rotation

In the four quadrants (upper right, upper left, lower left, and lower right) the signs indicate the direction of the vertical (j) component. The + sign indicates a vertically upward direction from the X axis and the - sign indicates a vertically downward direction from the X axis.

Consider the following example: the number, +4, in Figure 59-26A, indicates that 4 units are measured from the origin along the X axis in the positive direction. A +j operator placed before the 4 indicates that the number is to be orientated 90° counterclockwise and will now be measured along the Y axis in a positive direction. Likewise, a -j operator placed before the 4 indicates that the number is to be rotated 90° clockwise, and will now be measured along the Y axis in the negative di-rection.

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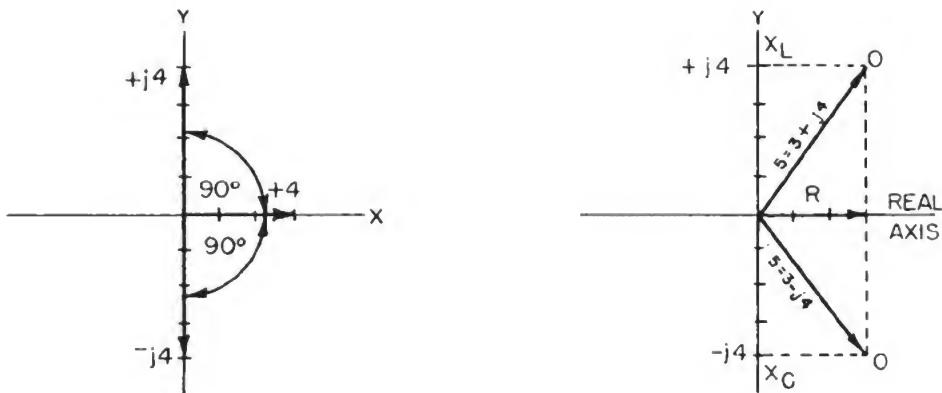
Figure 59-26 - Coordinates.

It may be recalled that inductive reactance, X_L, is indicated as lying along the Y axis in the positive direction, and capacitance reactance, X_C, is indicated as lying along the Y axis in the negative direction; resistance in each case is measured along the X axis in the positive direction. Therefore, +j has a direct association with X_L in that both are measured in the same direction along the Y axis, and -j similarly has a direct association with X_C.

The function of the j operator may be shown as follows: the expression 4 ohms, indicates that pure resistance is involved. In order to indicate that the 4 ohms represent capacitive reactance or inductive reactance a special symbol is needed. The use of the j operator gives a clear indication of the type of reactance. For example, if the j operator is not used, the 4 ohms is resistive. If +j is used (+j4), the 4 ohms is inductive reactance. If -j is used (-j4), the 4 ohms is capacitive reactance (Figure 59-26B).

The so-called COMPLEX NUMBER contains the "real" and the "imaginary" terms connected by a plus or a minus sign. Thus, 3+j4 and 3-j4 are complex numbers. This means that the 3 and the 4 in each instance are to be added vectorially, and the +j and -j indicates the direction of rotation of the vector following it. The real number in these examples is 3 and could be represented by a line drawn three units out from the origin on the positive X (resistance) axis. The imaginary number, +j4, could likewise be represented by a line extended 4 units from the origin on the positive Y, or X_L, axis; and -j4 could be represented by a line extended 4 units from the origin on the negative Y, or X_C, axis. The IMAGINARY, or QUADRATURE, quantities (for example, the X_L and X_C values) are always assumed to be drawn along the Y axis, and the REAL quantities (for example, the R values) are always assumed to be drawn along the X axis.

59-44. Addition and Subtraction of Complex Numbers

Values that are at right angles to each other cannot be added or subtracted in the usual sense of the word. Their sum or difference can only be indicated, as is done in the case of binomials (an expression involving two terms). Thus, assume that it is desired to add 3+4j to 3-j4.

$$
\begin{array}{r}
3+j4 \\
3-j4 \\
\hline
6-0
\end{array}
$$

The imaginary term disappears, and only the real term, 6, remains. If 3+j4 is added to 3+j4, the sum is the complex quantity, 6+j8.

One complex expression may also be subtracted from another complex expression in the same manner that binomials are treated. For example, 3-j2 may be subtracted from 3+j4 as:

$$
\begin{array}{r}
3+j4 \\
(-)\underline{3-j2} \\
0+j6
\end{array}
$$

The real term disappears, and the result is 6 units measured upward from the origin on the Y axis. If 3-j2 is subtracted from 6+j4, the difference is the complex quantity, 3+j6.

59-45. Multiplication and Division of Complex Numbers

Complex numbers are multiplied the same way that binomials are multiplied, for example, if 3-j2 is multiplied by 6+j3,

$$
\begin{array}{r}
3-j2 \\
\underline{6+j3} \\
18-j12 \\
\underline{+j\ 9-j^2 6} \\
18-j\ 3-j^2 6
\end{array}
$$

Because $j^2 = -1$, the product becomes $18-j3-(-1)6$, or $24-j3$.

Complex numbers may be divided in the same way that binomials containing a radical in the denominator are divided. The denominator is rationalized (multiplied by its conjugate, a term that is the same as the denominator except that it has the opposite algebraic sign before the j term), and the quotient is expressed as a term having only a real number as the divisor. For example, if 4+j3 is divided by 2-j2,

$$
\frac{4+j3}{2-j2} = \frac{(4+j3)(2+j2)}{(2-j2)(2+j2)} = \frac{2+j14}{8}
$$

$$
= \frac{1+j7}{4} = 0.25+j1.75
$$

59-46. Rectangular and Polar Forms

Sometimes it is more convenient to employ polar coordinates than rectangular coordinates. In RECTANGULAR FORM the vector is described in terms of the two sides of a right triangle, the hypotenuse of which is the vector. Thus, in Figure 59-27, vector OB is described in rectangular form by the complex number 3+j4. In other words, the end of the vector, OB, is 3 units along the +X axis and 4 units along the +Y axis, and its length is 5 units.

The vector, OB, may also be described if its length and the angle, θ, are given. When a vector is described by means of its magnitude and the angle it makes with the reference line it is expressed in the POLAR FORM. In this instance the length is 5 units and the angle, θ, is approximately 53°. The vector, OB, may then be expressed in the polar form as $5\underline{/+53^\circ}$.

The plus sign is shown with positive angles in this chapter in order to emphasize positive angles as contrasted with negative angles. The negative sign preceding the angle indicates clockwise rotation of the vector from the zero position.

59-47. Converting From One Form to the Other

Assume that the rectangular form is expressed by the complex number, 3+j4. The angle, θ, and the actual length of the vector, OB, are not given. The length, OB, can be determined by the use of the Pythagorean theorem (OB $= \sqrt{3^2+4^2}$), but it is usually simpler to determine first the angle, θ, by
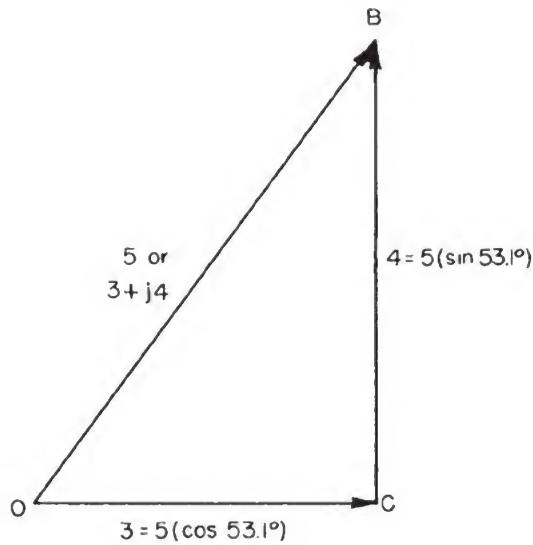
Figure 59-27 - Rectangular and polar forms.

finding the angle whose tangent is $\frac{4}{3}$ = 1.33. The angle is 53.1° from a table of trigonometric func-
tions. From the same table sin 53.1° = 0.8. Since sin θ = $\frac{BC}{OB}$, it follows that:

$$OB = \frac{BC}{\sin 53.1°} = \frac{4}{0.8} = 5;$$

and the vector may be expressed in the polar form as $5\underline{/+53.1°}$.

If the vector is originally expressed in the polar form as $5\underline{/+53.1°}$, it may be converted to the rec-
tangular form by the use of cos 53.1° and sin 53.1°. In this instance the vector is 5 units in length
and makes an angle of approximately 53.1° with the +X axis. Thus,

$$\sin 53.1° = \frac{BC}{5}$$

or   BC = 5 sin 53.1° = 5x0.8 = 4; cos 53.1° = $\frac{OC}{5}$,

or   OC = 5 cos 53.1° = 5x0.6 = 3.

Therefore, with BC and OC known, the vector may be expressed as the complex number 3+j4 (Figure
59-27).

The polar form may be converted to the rectangular form more concisely in the following manner:

$$5\underline{/+53.1°} = 5 \cos 53.1° + j5 \sin 53.1°$$

$$= (5x0.6) + (j5x0.8)$$

$$= 3+j4$$

59-48. Addition and Subtraction of Polar Vectors

Unless polar vectors are parallel to each other they cannot be added or subtracted algebraically. Therefore, the polar form is converted first to the rectangular form. Then the real components are added algebraically, and likewise, the imaginary components are added algebraically. Finally the result may be converted back to the polar form. Vector summation is indicated by the symbol + .

As an example, find the resultant vector, OR (Figure 59-28) of vectors OA and OB when OA=$10\underline{/30^o}$ and OB=$8\underline{/60^o}$. OR=OA + OB

Converting to rectangular form:

$$OA = 10 \cos 30^o + j10 \sin 30^o = 8.66 + j5.0$$
$$OB = 8 \cos 60^o + j 8 \sin 60^o = 4.0 + j6.93$$

Adding, like components, OR = 12.66 + j11.93.

Converting to polar form, OR = $12.66^2 + 11.93^2$

$$= 17.4 \text{ and } \tan \theta = \frac{11.93}{12.66} = 0.943$$

from which $\theta = 43.4^o$.
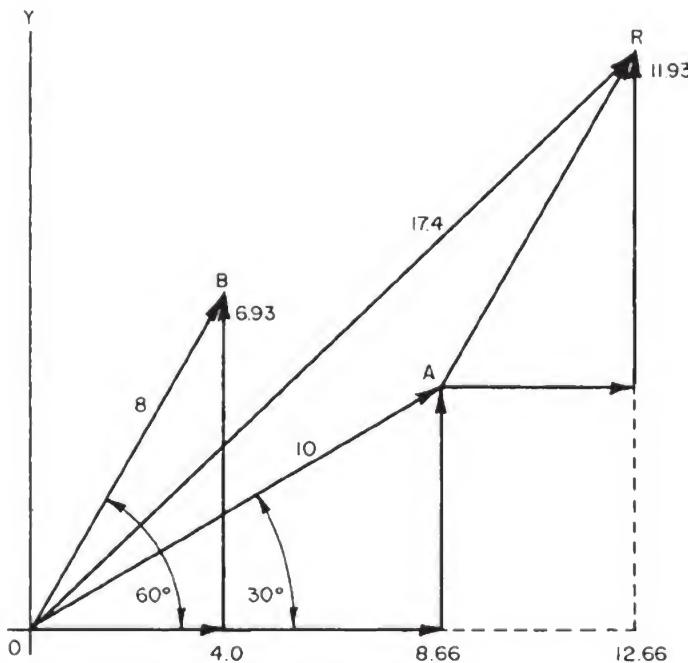
Figure 59-28 - Addition of vectors.

59-49. Multiplication and Division of Polar Vectors

The method of multiplying and dividing complex numbers by treating them as binomials and rationalizing the denominators may be simplified considerably by first converting the vectors into polar form and then proceeding to combine them in the following manner:

To obtain the product of two vectors, multiply the numbers representing the vectors in polar form and add their corresponding angles algebraically. The resultant vector is in polar form. Thus,

$$(5\underline{/+53^o})\ (5\underline{/-53^o}) = 25\underline{/0^o}$$

To obtain the quotient of two vectors, divide the numerator by the denominator as in ordinary division, then subtract algebraically the angle of the denominator from the angle in the numerator. The resultant vector is in polar form. Thus,

$$\frac{10\underline{/+25^o}}{5\underline{/-20^o}} = 2\underline{/45^o}$$

EXERCISE 37:

1. Express a 270° negative rotation using the (j) notation.

2. Express a 90° positive vector rotation using the (j) notation.

3. Add the quantities $(3-j7), (j6+4)$.

4. Divide the following quantities $(6 + j6) \div (7 - j7)$.

5. Perform the indicated operations $(5\underline{/-63^o}) + (11\underline{/31^o}) \times (3 - j6) \div (4\underline{/16^o})$.

N	0	1	2	3	4	5	6	7	8	9
0	....	0000	3010	4771	6021	6990	7782	8451	9031	9542
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067

| 2 | 3 | 4 | 9 |

N	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039

3   4

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	func-tion	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
deg	func-tion	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	func-tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°
28	sin	0. 4695	0. 4710	0. 4726	0. 4741	0. 4756	0. 4772	0. 4787	0. 4802	0. 4818	0. 4833
	cos	0. 8829	0. 8821	0. 8813	0. 8805	0. 8796	0. 8788	0. 8780	0. 8771	0. 8763	0. 8755
	tan	0. 5317	0. 5340	0. 5362	0. 5384	0. 5407	0. 5430	0. 5452	0. 5475	0. 5498	0. 5520
29	sin	0. 4848	0. 4863	0. 4879	0. 4894	0. 4909	0. 4924	0. 4939	0. 4955	0. 4970	0. 4985
	cos	0. 8746	0. 8738	0. 8729	0. 8721	0. 8712	0. 8704	0. 8695	0. 8686	0. 8678	0. 8669
	tan	0. 5543	0. 5566	0. 5589	0. 5612	0. 5635	0. 5658	0. 5681	0. 5704	0. 5727	0. 5750
30	sin	0. 5000	0. 5015	0. 5030	0. 5045	0. 5060	0. 5075	0. 5090	0. 5105	0. 5120	0. 5135
	cos	0. 8660	0. 8652	0. 8643	0. 8634	0. 8625	0. 8616	0. 8607	0. 8599	0. 8590	0. 8581
	tan	0. 5774	0. 5797	0. 5820	0. 5844	0. 5867	0. 5890	0. 5914	0. 5938	0. 5961	0. 5985
31	sin	0. 5150	0. 5165	0. 5180	0. 5195	0. 5210	0. 5225	0. 5240	0. 5255	0. 5270	0. 5284
	cos	0. 8572	0. 8563	0. 8554	0. 8545	0. 8536	0. 8526	0. 8517	0. 8508	0. 8499	0. 8490
	tan	0. 6009	0. 6032	0. 6056	0. 6080	0. 6104	0. 6128	0. 6152	0. 6176	0. 6200	0. 6224
32	sin	0. 5299	0. 5314	0. 5329	0. 5344	0. 5358	0. 5373	0. 5388	0. 5402	0. 5417	0. 5432
	cos	0. 8480	0. 8471	0. 8462	0. 8453	0. 8443	0. 8434	0. 8425	0. 8415	0. 8406	0. 8396
	tan	0. 6249	0. 6273	0. 6297	0. 6322	0. 6346	0. 6371	0. 6395	0. 6420	0. 6445	0. 6469
33	sin	0. 5446	0. 5461	0. 5476	0. 5490	0. 5505	0. 5519	0. 5534	0. 5548	0. 5563	0. 5577
	cos	0. 8387	0. 8377	0. 8368	0. 8358	0. 8348	0. 8339	0. 8329	0. 8320	0. 8310	0. 8300
	tan	0. 6494	0. 6519	0. 6544	0. 6569	0. 6594	0. 6619	0. 6644	0. 6669	0. 6694	0. 6720
34	sin	0. 5592	0. 5606	0. 5621	0. 5635	0. 5650	0. 5664	0. 5678	0. 5693	0. 5707	0. 5721
	cos	0. 8290	0. 8281	0. 8271	0. 8261	0. 8251	0. 8241	0. 8231	0. 8221	0. 8211	0. 8202
	tan	0. 6745	0. 6771	0. 6796	0. 6822	0. 6847	0. 6873	0. 6899	0. 6924	0. 6950	0. 6976
35	sin	0. 5736	0. 5750	0. 5764	0. 5779	0. 5793	0. 5807	0. 5821	0. 5835	0. 5850	0. 5864
	cos	0. 8192	0. 8181	0. 8171	0. 8161	0. 8151	0. 8141	0. 8131	0. 8121	0. 8111	0. 8100
	tan	0. 7002	0. 7028	0. 7054	0. 7080	0. 7107	0. 7133	0. 7159	0. 7186	0. 7212	0. 7239
36	sin	0. 5878	0. 5892	0. 5906	0. 5920	0. 5934	0. 5948	0. 5962	0. 5976	0. 5990	0. 6004
	cos	0. 8090	0. 8080	0. 8070	0. 8059	0. 8049	0. 8039	0. 8028	0. 8018	0. 8007	0. 7997
	tan	0. 7265	0. 7292	0. 7319	0. 7346	0. 7373	0. 7400	0. 7427	0. 7454	0. 7481	0. 7508
37	sin	0. 6018	0. 6032	0. 6046	0. 6060	0. 6074	0. 6088	0. 6101	0. 6115	0. 6129	0. 6143
	cos	0. 7986	0. 7976	0. 7965	0. 7955	0. 7944	0. 7934	0. 7923	0. 7912	0. 7902	0. 7891
	tan	0. 7536	0. 7563	0. 7590	0. 7618	0. 7646	0. 7673	0. 7701	0. 7729	0. 7757	0. 7785
38	sin	0. 6157	0. 6170	0. 6184	0. 6198	0. 6211	0. 6225	0. 6239	0. 6252	0. 6266	0. 6280
	cos	0. 7880	0. 7869	0. 7859	0. 7848	0. 7837	0. 7826	0. 7815	0. 7804	0. 7793	0. 7782
	tan	0. 7813	0. 7841	0. 7869	0. 7898	0. 7926	0. 7954	0. 7983	0. 8012	0. 8040	0. 8069
39	sin	0. 6293	0. 6307	0. 6320	0. 6334	0. 6347	0. 6361	0. 6374	0. 6388	0. 6401	0. 6414
	cos	0. 7771	0. 7760	0. 7749	0. 7738	0. 7727	0. 7716	0. 7705	0. 7694	0. 7683	0. 7672
	tan	0. 8098	0. 8127	0. 8156	0. 8185	0. 8214	0. 8243	0. 8273	0. 8302	0. 8332	0. 8361
40	sin	0. 6428	0. 6441	0. 6455	0. 6468	0. 6481	0. 6494	0. 6508	0. 6521	0. 6534	0. 6547
	cos	0. 7660	0. 7649	0. 7638	0. 7627	0. 7615	0. 7604	0. 7593	0. 7581	0. 7570	0. 7559
	tan	0. 8391	0. 8421	0. 8451	0. 8481	0. 8511	0. 8541	0. 8571	0. 8601	0. 8632	0. 8662
41	sin	0. 6561	0. 6574	0. 6587	0. 6600	0. 6613	0. 6626	0. 6639	0. 6652	0. 6665	0. 6678
	cos	0. 7547	0. 7536	0. 7524	0. 7513	0. 7501	0. 7490	0. 7478	0. 7466	0. 7455	0. 7443
	tan	0. 8693	0. 8724	0. 8754	0. 8785	0. 8816	0. 8847	0. 8878	0. 8910	0. 8941	0. 8972
deg	func-tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°

deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

deg	function	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°
56	sin	0. 8290	0. 8300	0. 8310	0. 8320	0. 8329	0. 8339	0. 8348	0. 8358	0. 8368	0. 8377
	cos	0. 5592	0. 5577	0. 5563	0. 5548	0. 5534	0. 5519	0. 5505	0. 5490	0. 5476	0. 5461
	tan	1. 4826	1. 4882	1. 4938	1. 4994	1. 5051	1. 5108	1. 5166	1. 5224	1. 5282	1. 5340
57	sin	0. 8387	0. 8396	0. 8406	0. 8415	0. 8425	0. 8434	0. 8443	0. 8453	0. 8462	0. 8471
	cos	0. 5446	0. 5432	0. 5417	0. 5402	0. 5388	0. 5373	0. 5358	0. 5344	0. 5329	0. 5314
	tan	1. 5399	1. 5458	1. 5517	1. 5577	1. 5637	1. 5697	1. 5757	1. 5818	1. 5880	1. 5941
58	sin	0. 8480	0. 8490	0. 8499	0. 8508	0. 8517	0. 8526	0. 8536	0. 8545	0. 8554	0. 8563
	cos	0. 5299	0. 5284	0. 5270	0 5255	0. 5240	0. 5225	0. 5210	0. 5195	0. 5180	0. 5165
	tan	1. 6003	1. 6066	1. 6128	1. 6191	1. 6255	1. 6319	1. 6383	1. 6447	1. 6512	1. 6577
59	sin	0. 8572	0. 8581	0. 8590	0. 8599	0. 8607	0. 8616	0. 8625	0. 8634	0. 8643	0. 8652
	cos	0. 5150	0. 5135	0. 5120	0. 5105	0. 5090	0. 5075	0. 5060	0. 5045	0. 5030	0. 5015
	tan	1. 6643	1. 6709	1. 6775	1. 6842	1. 6909	1. 6977	1. 7045	1. 7113	1. 7182	1. 7251
60	sin	0. 8660	0. 8669	0. 8678	0. 8686	0. 8695	0. 8704	0. 8712	0. 8721	0. 8729	0. 8738
	cos	0. 5000	0. 4985	0. 4970	0. 4955	0. 4939	0. 4924	0. 4909	0. 4894	0. 4879	0. 4863
	tan	1. 7321	1. 7391	1. 7461	1. 7532	1. 7603	1. 7675	1. 7747	1. 7820	1. 7893	1. 7966
61	sin	0. 8746	0. 8755	0. 8763	0. 8771	0. 8780	0. 8788	0. 8796	0. 8805	0. 8813	0. 8821
	cos	0. 4848	0. 4833	0. 4818	0. 4802	0. 4787	0. 4772	0. 4756	0. 4741	0. 4726	0. 4710
	tan	1. 8040	1. 8115	1. 8190	1. 8265	1. 8341	1. 8418	1. 8495	1. 8572	1. 8650	1. 8728
62	sin	0. 8829	0. 8838	0. 8846	0. 8854	0. 8862	0. 8870	0. 8878	0. 8886	0. 8894	0. 8902
	cos	0. 4695	0. 4679	0. 4664	0. 4648	0. 4633	0. 4617	0. 4602	0. 4586	0. 4571	0. 4555
	tan	1. 8807	1. 8887	1. 8967	1. 9047	1. 9128	1. 9210	1. 9292	1. 9375	1. 9458	1. 9542
63	sin	0. 8910	0. 8918	0. 8926	0. 8934	0. 8942	0. 8949	0. 8957	0. 8965	0. 8973	0. 8980
	cos	0. 4540	0. 4524	0. 4509	0. 4493	0. 4478	0. 4462	0. 4446	0. 4431	0. 4415	0. 4399
	tan	1. 9626	1. 9711	1. 9797	1. 9883	1. 9970	2. 0057	2. 0145	2. 0233	2. 0323	2. 0413
64	sin	0. 8988	0. 8996	0. 9003	0. 9011	0. 9018	0. 9026	0. 9033	0. 9041	0. 9048	0. 9056
	cos	0. 4384	0. 4368	0. 4352	0. 4337	0. 4321	0. 4305	0. 4289	0. 4274	0. 4258	0. 4242
	tan	2. 0503	2. 0594	2. 0686	2. 0778	2. 0872	2. 0965	2. 1060	2. 1155	2. 1251	2. 1348
65	sin	0. 9063	0. 9070	0. 9078	0. 9085	0. 9092	0. 9100	0. 9107	0. 9114	0. 9121	0. 9128
	cos	0. 4226	0. 4210	0. 4195	0. 4179	0. 4163	0. 4147	0. 4131	0. 4115	0. 4099	0. 4083
	tan	2. 1445	2. 1543	2. 1642	2. 1742	2. 1842	2. 1943	2. 2045	2. 2148	2. 2251	2. 2355
66	sin	0. 9135	0. 9143	0. 9150	0. 9157	0. 9164	0. 9171	0. 9178	0. 9184	0. 9191	0. 9198
	cos	0. 4067	0. 4051	0. 4035	0. 4019	0. 4003	0. 3987	0. 3971	0. 3955	0. 3939	0. 3923
	tan	2. 2460	2. 2566	2. 2673	2. 2781	2. 2889	2. 2998	2. 3109	2. 3220	2. 3332	2. 3445
67	sin	0. 9205	0. 9212	0. 9219	0. 9225	0. 9232	0. 9239	0. 9245	0. 9252	0. 9259	0. 9265
	cos	0. 3907	0. 3891	0. 3875	0. 3859	0. 3843	0. 3827	0. 3811	0. 3795	0. 3778	0. 3762
	tan	2. 3559	2. 3673	2. 3789	2. 3906	2. 4023	2. 4142	2. 4262	2. 4383	2. 4504	2. 4627
68	sin	0. 9272	0. 9278	0. 9285	0. 9291	0. 9298	0. 9304	0. 9311	0. 9317	0. 9323	0. 9330
	cos	0. 3746	0. 3730	0. 3714	0. 3697	0. 3681	0. 3665	0. 3649	0. 3633	0. 3616	0. 3600
	tan	2. 4751	2. 4876	2. 5002	2. 5129	·2. 5257	2. 5386	2. 5517	2. 5649	2. 5782	2. 5916
69	sin	0. 9336	0.9342	0. 9348	0. 9354	0. 9361	0. 9367	0. 9373	0. 9379	0. 9385	0. 9391
	cos	0. 3584	0. 3567	0. 3551	0. 3535	0. 3518	0. 3502	0. 3486	0. 3469	0. 3453	0. 3437
	tan	2. 6051	2. 6187	2. 6325	2. 6464	2. 6605	2. 6746	2. 6889	2. 7034	2. 7179	2. 7326
deg	function	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°

deg	function	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°
70	sin	0. 9397	0. 9403	0. 9409	0. 9415	0. 9421	0. 9426	0. 9432	0. 9438	0. 9444	0. 9449
	cos	0. 3420	0. 3404	0. 3387	0. 3371	0. 3355	0. 3338	0. 3322	0. 3305	0. 3289	0. 3272
	tan	2. 7475	2. 7625	2. 7776	2. 7929	2. 8083	2. 8239	2. 8397	2. 8556	2. 8716	2. 8878
71	sin	0. 9455	0. 9461	0. 9466	0. 9472	0. 9478	0. 9483	0. 9489	0. 9494	0. 9500	0. 9505
	cos	0. 3256	0. 3239	0. 3223	0. 3206	0. 3190	0. 3173	0. 3156	0. 3140	0. 3123	0. 3107
	tan	2. 9042	2. 9208	2. 9375	2. 9544	0. 9714	2. 9887	3. 0061	3. 0237	3. 0415	3. 0595
72	sin	0. 9511	0. 9516	0. 9521	0. 9527	0. 9532	0. 9537	0. 9542	0. 9548	0. 9553	0. 9558
	cos	0. 3090	0. 3074	0. 3057	0. 3040	0. 3024	0. 3007	0. 2990	0. 2974	0. 2957	0. 2940
	tan	3. 0777	3. 0961	3. 1146	3. 1334	3. 1524	3. 1716	3. 1910	3. 2106	3. 2305	3. 2506
73	sin	0. 9563	0. 9568	0. 9573	0. 9578	0. 9583	0. 9588	0. 9593	0. 9598	0. 9603	0. 9608
	cos	0. 2924	0. 2907	2. 2890	0. 2874	0. 2857	0. 2840	0. 2823	0. 2807	0. 2790	0. 2773
	tan	3. 2709	3. 2914	3. 3122	3. 3332	3. 3544	3. 3759	3. 3977	3. 4197	3. 4420	3. 4646
74	sin	0. 9613	0. 9617	0. 9622	0. 9627	0. 9632	0. 9636	0. 9641	0. 9646	0. 9650	0. 9655
	cos	0. 2756	0. 2740	0. 2723	0. 2706	0. 2689	0. 2672	0. 2656	0. 2639	0. 2622	0. 2605
	tan	3. 4874	3. 5105	3. 5339	3. 5576	3. 5816	3. 6059	3. 6305	3. 6554	3. 6806	3. 7062
75	sin	0. 9659	0. 9664	0. 9668	0. 9673	0. 9677	0. 9681	0. 9686	0. 9690	0. 9694	0. 9699
	cos	0. 2588	0. 2571	0. 2554	0. 2538	0. 2521	0. 2504	0. 2487	0. 2470	0. 2453	0. 2436
	tan	3. 7321	3. 7583	3. 7848	3. 8118	3. 8391	3. 8667	3. 8947	3. 9232	3. 9520	3. 9812
76	sin	0. 9703	0. 9707	0. 9711	0. 9715	0. 9720	0. 9724	0. 9728	0. 9732	0. 9736	0. 9740
	cos	0. 2419	0. 2402	0. 2385	0. 2368	0. 2351	0. 2334	0. 2317	0. 2300	0. 2284	0. 2267
	tan	4. 0108	4. 0408	4. 0713	4. 1022	4. 1335	4. 1653	4. 1976	4. 2303	4. 2635	4. 2972
77	sin	0. 9744	0. 9748	0. 9751	0. 9755	0. 9759	0. 9763	0. 9767	0. 9770	0. 9774	0. 9778
	cos	0. 2250	0. 2232	0. 2215	0. 2198	0. 2181	0. 2164	0. 2147	0. 2130	0. 2113	0. 2096
	tan	4. 3315	4. 3662	4. 4015	4. 4374	4. 4737	4. 5107	4. 5483	4. 5864	4. 6252	4. 6646
78	sin	0. 9781	0. 9785	0. 9789	0. 9792	0. 9796	0. 9799	0. 9803	0. 9806	0. 9810	0. 9813
	cos	0. 2079	0. 2062	0. 2045	0. 2028	0. 2011	0. 1994	0. 1977	0. 1959	0. 1942	0. 1925
	tan	4. 7046	4. 7453	4. 7867	4. 8288	4. 8716	4. 9152	4. 9594	5. 0045	5. 0504	5. 0970
79	sin	0. 9816	0. 9820	0. 9823	0. 9826	0. 9829	0. 9833	0. 9836	0. 9839	0. 9842	0. 9845
	cos	0. 1908	0. 1891	0. 1874	0. 1857	0. 1840	0. 1822	0. 1805	0. 1788	0. 1771	0. 1754
	tan	5. 1446	5. 1929	5. 2422	5. 2924	5. 3435	5. 3955	5. 4486	5. 5026	5. 5578	5. 6140
80	sin	0. 9848	0. 9851	0. 9854	0. 9857	0. 9860	0. 9863	0. 9866	0. 9869	0. 9871	0. 9874
	cos	0. 1736	0. 1719	0. 1702	0. 1685	0. 1668	0. 1650	0. 1633	0. 1616	0. 1599	0. 1582
	tan	5. 6713	5. 7297	5. 7894	5. 8502	5. 9124	5. 9758	6. 0405	6. 1066	6. 1742	6. 2432
81	sin	0. 9877	0. 9880	0. 9882	0. 9885	0. 9888	0. 9890	0. 9893	0. 9895	0. 9898	0. 9900
	cos	0. 1564	1. 1547	0. 1530	0. 1513	0. 1495	0. 1478	0. 1461	0. 1444	0. 1426	0. 1409
	tan	6. 3138	6. 3859	6. 4596	6. 5350	6. 6122	6. 6912	6. 7720	6. 8548	6. 9395	7. 0264
82	sin	0. 9903	0. 9905	0. 9907	0. 9910	0. 9912	0. 9914	0. 9917	0. 9919	0. 9921	0. 9923
	cos	0. 1392	0. 1374	0. 1357	0. 1340	0. 1323	0. 1305	0. 1288	0. 1271	0. 1253	0. 1236
	tan	7. 1154	7. 2066	7. 3002	7. 3962	7. 4947	7. 5958	7. 6996	7. 8062	7. 9158	8. 0285
83	sin	0. 9925	0. 9928	0. 9930	0. 9932	0. 9934	0. 9936	0. 9938	0. 9940	0. 9942	0. 9943
	cos	0. 1219	0. 1201	0. 1184	0. 1167	0. 1149	0. 1132	0. 1115	0. 1097	0. 1080	0. 1063
	tan	8. 1443	8. 2636	8. 3863	8. 5126	8. 6427	8. 7769	8. 9152	9. 0579	9. 2052	9. 3572
deg	function	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°

deg	func-tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°
84	sin	0. 9945	0. 9947	0. 9949	0. 9951	0. 9952	0. 9954	0. 9956	0. 9957	0. 9959	0. 9960
	cos	0. 1045	0. 1028	0. 1011	0. 0993	0. 0976	0. 0958	0. 0941	0. 0924	0. 0906	0. 0889
	tan	9. 5144	9. 6768	9. 8448	10. 02	10. 20	10. 39	10. 58	10. 78	10. 99	11. 20
85	sin	0. 9962	0. 9963	0. 9965	0. 9966	0. 9968	0. 9969	0. 9971	0. 9972	0. 9973	0. 9974
	cos	0. 0872	0. 0854	0. 0837	0. 0819	0. 0802	0. 0785	0. 0767	0. 0750	0. 0732	0. 0715
	tan	11. 43	11. 66	11. 91	12. 16	12. 43	12. 71	13. 00	13. 30	13. 62	13. 95
86	sin	0. 9976	0. 9977	0. 9978	0. 9979	0. 9980	0. 9981	0. 9982	0. 9983	0. 9984	0. 9985
	cos	0. 0698	0. 0680	0. 0663	0. 0645	0. 0628	0. 0610	0. 0593	0. 0576	0. 0558	0. 0541
	tan	14. 30	14. 67	15. 06	15. 46	15. 89	16. 35	16. 83	17. 34	17. 89	18. 46
87	sin	0. 9986	0. 9987	0. 9988	0. 9989	0. 9990	0. 9990	0. 9991	0. 9992	0. 9993	0. 9993
	cos	0. 0523	0. 0506	0. 0488	0. 0471	0. 0454	0. 0436	0. 0419	0. 0401	0. 0384	0. 0366
	tan	19. 08	19. 74	20. 45	21. 20	22. 02	22. 90	23. 86	24. 90	26. 03	27. 27
88	sin	0. 9994	0. 9995	0. 9995	0. 9996	0. 9996	0. 9997	0. 9997	0. 9997	0. 9998	0. 9998
	cos	0. 0349	0. 0332	0. 0314	0. 0297	0. 0279	0. 0262	0. 0244	0. 0227	0. 0209	0. 0192
	tan	28. 64	30. 14	31. 82	33. 69	35. 80	38. 19	40. 92	44. 07	47. 74	52. 08
89	sin	0. 9998	0. 9999	0. 9999	0. 9999	0. 9999	1. 000	1. 000	1. 000	1. 000	1. 000
	cos	0. 0175	0. 0157	0. 0140	0. 0122	0. 0105	0. 0087	0. 0070	0. 0052	0. 0035	0. 0017
	tan	57. 29	63. 66	71. 62	81. 85	95. 49	114. 6	143. 2	191. 0	286. 5	573. 0
deg	func-tion	0. 0°	0. 1°	0. 2°	0. 3°	0. 4°	0. 5°	0. 6°	0. 7°	0. 8°	0. 9°

Appendix III

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

No.	Square	Cube	Square Root	Cube Root
1	1	1	1.0000	1.0000
2	4	8	1.4142	1.2599
3	9	27	1.7321	1.4423
4	16	64	2.0000	1.5874
5	25	125	2.2361	1.7100
6	36	216	2.4495	1.8171
7	49	343	2.6458	1.9129
8	64	512	2.8284	2.0000
9	81	729	3.0000	2.0801
10	100	1000	3.1623	2.1544
11	121	1331	3.3166	2.2240
12	144	1728	3.4641	2.2894
13	169	2197	3.6056	2.3513
14	196	2744	3.7417	2.4101
15	225	3375	3.8730	2.4662
16	256	4096	4.0000	2.5198
17	289	4913	4.1231	2.5713
18	324	5832	4.2426	2.6207
19	361	6859	4.3589	2.6684
20	400	8000	4.4721	2.7144
21	441	9261	4.5826	2.7589
22	484	10648	4.6904	2.8020
23	529	12167	4.7958	2.8439
24	576	13824	4.8990	2.8845
25	625	15625	5.0000	2.9240
26	676	17576	5.0990	2.9625
27	729	19683	5.1962	3.0000
28	784	21952	5.2915	3.0366
29	841	24389	5.3852	3.0723
30	900	27000	5.4772	3.1072
31	961	29791	5.5678	3.1414
32	1024	32768	5.6569	3.1748
33	1089	35937	5.7446	3.2075
34	1156	39304	5.8310	3.2396
35	1225	42875	5.9161	3.2711
36	1296	46656	6.0000	3.3019
37	1369	50653	6.0828	3.3322
38	1444	54872	6.1644	3.3620
39	1521	59319	6.2450	3.3912

No.	Square	Cube	Square Root	Cube Root
40	1600	64000	6.3246	3.4200
41	1681	68921	6.4031	3.4482
42	1764	74088	6.4807	3.4760
43	1849	79507	6.5574	3.5034
44	1936	85184	6.6332	3.5303
45	2025	91125	6.7082	3.5569
46	2116	97336	6.7823	3.5830
47	2209	103823	6.8557	3.6088
48	2304	110592	6.9282	3.6342
49	2401	117649	7.0000	3.6593
50	2500	125000	7.0711	3.6840
51	2601	132651	7.1414	3.7084
52	2704	140608	7.2111	3.8325
53	2809	148877	7.2801	3.7563
54	2916	157464	7.3485	3.7798
55	3025	166375	7.4162	3.8030
56	3136	175616	7.4333	3.8259
57	3249	185193	7.5498	3.8485
58	3364	195112	7.6158	3.8709
59	3481	205379	7.6811	3.8930
60	3600	216000	7.7460	3.9149
61	3721	226981	7.8102	3.9365
62	3844	238328	7.8740	3.9579
63	3969	250047	7.9373	3.9791
64	4096	262114	8.0000	4.0000
65	4225	274625	8.0623	4.0207
66	4356	287496	8.1240	4.0412
67	4489	300763	8.1854	4.0615
68	4624	314432	8.2462	4.0817
69	4761	328509	8.3066	4.1016
70	4900	343000	8.3666	4.1213
71	5041	357911	8.4261	4.1408
72	5184	373248	8.4853	4.1602
73	5329	389017	8.5440	4.1793
74	5476	405224	8.6023	4.1983
75	5625	421875	8.6603	4.2172
76	5776	438976	8.7178	4.2358
77	5929	456533	8.7750	4.2543
78	6084	474552	8.8318	4.2727
79	6241	493039	8.8882	4.2908
80	6400	512000	8.9443	4.3089
81	6561	531441	9.0000	4.3267
82	6724	551368	9.0554	4.3445
83	6889	571787	9.1104	4.3621
84	7056	592704	9.1652	4.3795
85	7225	614125	9.2195	4.3968
86	7396	636056	9.2736	4.4140
87	7569	658503	9.3274	4.4310
88	7744	681472	9.3808	4.4480
89	7921	704969	9.4340	4.4647
90	8100	729000	9.4868	4.4814
91	8281	753571	9.5394	4.4979
92	8464	778688	9.5917	4.5144
93	8649	804357	9.6437	4.5307
94	8836	830584	9.6954	4.5468
95	9025	857375	9.7468	4.5629

No.	Square	Cube	Square Root	Cube Root
96	9216	884736	9.7980	4.5789
97	9409	912673	9.4889	4.5947
98	9604	941192	9.8995	4.6104
99	9801	970299	9.9499	4.6261
100	10000	1000000	10.0000	4.6416
101	10201	1030301	10.4099	4.6570
102	10404	1061208	10.0995	4.6723
103	10609	1092727	10.1489	4.6875
104	10816	1124864	10.1980	4.7027
105	11025	1157625	10.2470	4.7177
106	11236	1191016	10.2956	4.7326
107	11449	1225043	10.3441	4.7475
108	11664	1259712	10.3923	4.7622
109	11881	1295029	10.4403	4.7769
110	12100	1331000	10.4881	4.7914
111	12321	1367631	10.5357	4.8059
112	12544	1404928	10.5830	4.8203
113	12769	1442897	10.6301	4.8346
114	12996	1481544	10.6771	4.8488
115	13225	1520875	10.7238	4.8629
116	13456	1560896	10.7703	4.8770
117	13689	1601613	10.8167	4.8910
118	13924	1643032	10.8628	4.9049
119	14161	1685159	10.9087	4.9187
120	14400	1728000	10.9545	4.9324
121	14641	1771561	11.0000	4.9461
122	14884	1815848	11.0454	4.9597
123	15129	1860867	11.0905	4.9732
124	15376	1906624	11.1355	4.9866
125	15625	1953125	11.1803	5.0000
126	15876	2000376	11.2250	5.0133
127	16129	2048383	11.2694	5.0265
128	16384	2097152	11.3137	5.0397
129	16641	2146689	11.3578	5.0528
130	16900	2197000	11.4018	5.0658
131	17161	2248091	11.4455	5.0788
132	17424	2299968	11.4891	5.0916
133	17689	2352637	11.5326	5.1045
134	17956	2406104	11.5758	5.1172
135	18225	2460375	11.6190	5.1299
136	18496	2515456	11.6619	5.1426
137	18769	2571353	11.7047	5.1551
138	19044	2628072	11.7473	5.1676
139	19321	2685619	11.7898	5.1801
140	19600	2744000	11.8322	5.1925
141	19881	2803221	11.8743	5.2048
142	20164	2863288	11.9164	5.2171
143	20449	2924207	11.9583	5.2293
144	20736	2985984	12.0000	5.2415
145	21025	3048625	12.0416	5.2536
146	21316	3112136	12.0830	5.2656
147	21609	3176523	12.1244	5.2776
148	21904	3241792	12.1655	5.2896
149	22201	3307949	12.2066	5.3015

No.	Square	Cube	Square Root	Cube Root	No.	Square	Cube	Square Root	Cube Root
150	22500	3375000	12.2474	5.3133	204	41616	8489664	14.2829	5.8868
151	22801	3442951	12.2882	5.3251	205	42025	8615125	14.3178	5.8964
152	23104	3511808	12.3288	5.3368	206	42436	8741816	14.3527	5.9059
153	23409	3581577	12.3693	5.3485	207	42849	8869743	14.3875	5.9155
154	23716	3652264	12.4097	5.3601	208	43264	8998912	14.4222	5.9250
155	24025	3723875	12.4499	5.3717	209	43681	9129329	14.4568	5.9345
156	24336	3796416	12.4900	5.3832					
157	24649	3869893	12.5300	5.3947	210	44100	9261000	14.4914	5.9439
158	24964	3944312	12.5698	5.4061	211	44521	9393931	14.5258	5.9533
159	25281	4019679	12.6095	5.4175	212	44944	9528128	14.5602	5.9627
					213	45369	9663597	14.5945	5.9721
160	25600	4096000	12.6491	5.4288	214	45796	9800344	14.6287	5.9814
161	25921	4173281	12.6886	5.4401	215	46225	9938375	14.6629	5.9907
162	26244	4251528	12.7279	5.4514	216	46656	10077696	14.6969	6.0000
163	26569	4330747	12.7671	5.4626	217	47089	10218313	14.7309	6.0092
164	26896	4410944	12.8062	5.4737	218	47524	10360232	14.7648	6.0185
165	27225	4492125	12.8452	5.4848	219	47961	10503459	14.7986	6.0277
166	27556	4574296	12.8841	5.4959					
167	27889	4657463	12.9228	5.5069	220	48400	10648000	14.8324	6.0368
168	28224	4741632	12.9615	5.5178	221	48841	10793861	14.8661	6.0459
169	28561	4826809	13.0000	5.5288	222	49284	10941048	14.8997	6.0550
					223	49729	11089567	14.9332	6.0641
170	28900	4913000	13.0384	5.5397	224	50176	11239424	14.9666	6.0732
171	29241	5000211	13.0767	5.5505	225	50625	11390625	15.0000	6.0822
172	29584	5088448	13.1149	5.5613	226	51076	11543176	15.0333	6.0912
173	29929	5177717	13.1529	5.5721	227	51529	11697083	15.0665	6.1002
174	30276	5268024	13.1909	5.5828	228	51984	11852352	15.0997	6.1091
175	30625	5359375	13.2288	5.5934	229	52441	12008989	15.1327	6.1180
176	30976	5451776	13.2665	5.6041					
177	31329	5545233	13.3041	5.6147	230	52900	12167000	15.1658	6.1269
178	31684	5639752	13.3417	5.6252	231	53361	12326391	15.1987	6.1358
179	32041	5735339	13.3791	5.6357	232	53824	12487168	15.2315	6.1446
					233	54289	12649337	15.2643	6.1534
					234	54756	12812904	15.2971	6.1622
180	32400	5832000	13.4164	5.6462	235	55225	12977875	15.3297	6.1710
181	32761	5929741	13.4536	5.6567	236	55696	13144256	15.3623	6.1797
182	33124	6028568	13.4907	5.6671	237	56169	13312053	15.3948	6.1885
183	33489	6128487	13.5277	5.6774	238	56644	13481272	15.4272	6.1972
184	33856	6229504	13.5647	5.6877	239	57121	13651919	15.4596	6.2058
185	34225	6331625	13.6015	5.6980					
186	34596	6434856	13.6382	5.7083	240	57600	13824000	15.4919	6.2145
187	34969	6539203	13.6748	5.7185	241	58081	13997521	15.5242	6.2231
188	35344	6644672	13.7113	5.7287	242	58564	14172488	15.5563	6.2317
189	35721	6751269	13.7477	5.7388	243	59049	14348907	15.5885	6.2403
					244	59536	14526784	15.6205	6.2488
190	36100	6859000	13.7840	5.7489	245	60025	14706125	15.6525	6.2573
191	36481	6967871	13.8203	5.7590	246	60516	14886936	15.6844	6.2658
192	36864	7077888	13.8564	5.7690	247	61009	15069223	15.7162	6.2743
193	37249	7189057	13.8924	5.7790	248	61504	15252992	15.7480	6.2828
194	37636	7301384	13.9284	5.7890	249	62001	15438249	15.7797	6.2912
195	38025	7414875	13.9642	5.7989					
196	38416	7529536	14.0000	5.8088	250	62500	15625000	15.8114	6.2996
197	38809	7645373	14.0357	5.8186	251	63001	15813251	15.8430	6.3080
198	39204	7762392	14.0712	5.8285	252	63504	16003008	15.8745	6.3164
199	39601	7880599	14.1067	5.8383	253	64009	16194277	15.9060	6.3247
					254	64516	16387064	15.9374	6.3330
200	40000	8000000	14.1421	5.8480	255	65025	16581375	15.9687	6.3413
201	40401	8120601	14.1774	5.8578	256	65536	16777216	16.0000	6.3496
202	40804	8242408	14.2127	5.8675	257	66049	16974593	16.0312	6.3579
203	41209	8365427	14.2478	5.8771					

No.	Square	Cube	Square Root	Cube Root
258	66564	17173512	16.0624	6.3661
259	67081	17373979	16.0935	6.3743
260	67600	17576000	16.1245	6.3825
261	68121	17779581	16.1555	6.3907
262	68644	17984728	16.1864	6.3988
263	69169	18191447	16.2173	6.4070
264	69696	18399744	16.2481	6.4151
265	70225	18609625	16.2788	6.4232
266	70756	18821096	16.3095	6.4312
267	71289	19034163	16.3401	6.4393
268	71824	19248832	16.3707	6.4473
269	72361	19465109	16.4012	6.4553
270	72900	19683000	16.4317	6.4633
271	73441	19902511	16.4621	6.4713
272	73984	20123648	16.4924	6.4792
273	74529	20346417	16.5227	6.4872
274	75076	20570824	16.5529	6.4951
275	75625	20796875	16.5831	6.5030
276	76176	21024576	16.6132	6.5108
277	76729	21253933	16.6433	6.5187
278	77284	21484952	16.6733	6.5265
279	77841	21717639	16.7033	6.5343
280	78400	21952000	16.7332	6.5421
281	78961	22188041	16.7631	6.5499
282	79524	22425768	16.7929	6.5577
283	80089	22665187	16.8226	6.5654
284	80656	22906304	16.8523	6.5731
285	81225	23149125	16.8819	6.5808
286	81796	23393656	16.9115	6.5885
287	82369	23639903	16.9411	6.5962
288	82944	23887872	16.9706	6.6039
289	83521	24137569	17.0000	6.6115
290	84100	24389000	17.0294	6.6191
291	84681	24642171	17.0587	6.6267
292	85264	24897088	17.0880	6.6343
293	85849	25153757	17.1172	6.6419
294	86436	25412184	17.1464	6.6494
295	87025	25672375	17.1756	6.6569
296	87616	25934336	17.2047	6.6644
297	88209	26198073	17.2337	6.6719
298	88804	26463592	17.2627	6.6794
299	89401	26730899	17.2916	6.6869
300	90000	27000000	17.3205	6.6943
301	90601	27270901	17.3494	6.7018
302	91204	27543608	17.3781	6.7092
303	91809	27818127	17.4069	6.7166
304	92416	28094464	17.4356	6.7240
305	93025	28372625	17.4642	6.7313
306	93636	28652616	17.4929	6.7387
307	94249	28934443	17.5214	6.7460
308	94864	29218112	17.5499	6.7533
309	95481	29503629	17.5784	6.7606

No.	Square	Cube	Square Root	Cube Root
310	96100	29791000	17.6068	6.7679
311	96721	30080231	17.6352	6.7752
312	97344	30371328	17.6635	6.7824
313	97969	30664297	17.6918	6.7897
314	98596	30959144	17.7200	6.7969
315	99225	31255875	17.7482	6.8041
316	99856	31554496	17.7764	6.8113
317	100489	31855013	17.8045	6.8185
318	101124	32157432	17.8326	6.8256
319	101761	32461759	17.8606	6.8328
320	102400	32768000	17.8885	6.8399
321	103041	33076161	17.9165	6.8470
322	103684	33386248	17.9444	6.8541
323	104329	33698267	17.9722	6.8612
324	104976	34012224	18.0000	6.8683
325	105625	34328125	18.0278	6.8753
326	106276	34645976	18.0555	6.8824
327	106929	34965783	18.0831	6.8894
328	107584	35287552	18.1108	6.8964
329	108241	35611289	18.1384	6.9034
330	108900	35937000	18.1659	6.9104
331	109561	36264691	18.1934	6.9174
332	110224	36594368	18.2209	6.9244
333	110889	36926037	18.2483	6.9313
334	111556	37259704	18.2757	6.9382
335	112225	37595375	18.3030	6.9451
336	112896	37933056	18.3303	6.9521
337	113569	38272753	18.3576	6.9589
338	114244	38614472	18.3848	6.9658
339	114921	38958219	18.4120	6.9727
340	115600	39304000	18.4391	6.9795
341	116281	39651821	18.4662	6.9864
342	116964	40001688	18.4932	6.9932
343	117649	40353607	18.5203	7.0000
344	118336	40707584	18.5472	7.0068
345	119025	41063625	18.5742	7.0136
346	119716	41421736	18.6011	7.0203
347	120409	41781923	18.6279	7.0271
348	121104	42144192	18.6548	7.0338
349	121801	42508549	18.6815	7.0406
350	122500	42875000	18.7083	7.0473
351	123201	43243551	18.7350	7.0540
352	123904	43614208	18.7617	7.0607
353	124609	43986977	18.7883	7.0674
354	125316	44361864	18.8149	7.0740
355	126025	44738875	18.8414	7.0807
356	126736	45118016	18.8680	7.0873
357	127449	45499293	18.8944	7.0940
358	128164	45882712	18.9209	7.1006
359	128881	46268279	18.9473	7.1072

No.	Square	Cube	Square Root	Cube Root	No.	Square	Cube	Square Root	Cube Root
360	129600	46656000	18.9737	7.1138					
361	130321	47045881	19.0000	7.1204	416	173056	71991296	20.3961	7.4650
362	131044	47437928	19.0263	7.1269	417	173889	72511713	20.4206	7 4710
363	131769	47832147	19.0526	7.1335	418	174724	73034632	20.4450	7.4770
364	132496	48228544	19.0788	7.1400	419	175561	73560059	20.4695	7.4829
365	133225	48627125	19.1050	7.1466					
366	133956	49027896	19.1311	7.1531	420	176400	74088000.	20.4939	7.4889
367	134689	49430863	19.1572	7.1596	421	177241	74618461	20.5183	7.4948
368	135424	49836032	19.1833	7.1661	422	178084	75151448	20.5426	7.5007
369	136161	50243409	19.2094	7.1726	423	178929	75686967	20.5670	7.5067
					424	179776	76225024	20.5913	7.5126
370	136900	50653000	19.2354	7.1791	425	180625	76765625	20.6155	7.5185
371	137641	51064811	19.2614	7.1855	426	181476	77308776	20.6398	7.5244
372	138384	51478848	19.2873	7.1920	427	182329	77854483	20.6640	7.5302
373	139129	51895117	19.3132	7.1984	428	183184	78402752	20.6882	7.5361
374	139876	52313624	19.3391	7.2048	429	184041	78953589	20.7123	7.5420
375	140625	52734375	19.3649	7.2112					
376	141376	53157376	19.3907	7.2177	430	184900	79507000	20.7364	7.5478
377	142129	53582633	19.4165	7.2240	431	185761	80062991	20.7605	7.5537
378	142884	54010152	19.4422	7.2304	432	186624	80621568	20.7846	7.5595
379	143641	54439939	19 4679	7.2368	433	187489	81182737	20.8087	7.5654
					434	188356	81746504	20.8327	7.5712
380	144400	54872000	19.4936	7.2432	435	189225	82312875	20.8567	7.5770
381	145161	55306341	19.5192	7.2495	436	190096	82881856	20.8806	7.5828
382	145924	55742963	19.5448	7.2558	437	190969	83453453	20.9045	7.5886
383	146689	56181887	19.5704	7.2622	438	191844	84027672	20.9284	7.5944
384	147456	56623104	19.5959	7.2685	439	192721	84604519	20.9523	7.6001
385	148225	57000625	19.6214	7.2748					
386	148996	57 2456	19.6469	7.2811	440	193600	85184000	20.9762	7.6059
387	149769	60603	19.6723	7.2874	441	194481	85766121	21.0000	7.6117
388	150544	411072	19.6977	7.2936	442	195364	86350888	21.0238	7.6174
389	151321	8863869	19.7231	7.2999	443	196249	86938307	21.0476	7.6232
					444	197136	87528384	21.0713	7.6289
390	152100	59319000	19.7484	7.3061	445	198025	88121125	21.0950	7.6346
391	152881	59776471	19.7737	7.3124	446	198916	88716536	21.1187	7.6403
392	153664	60236288	19.7990	7.3186	447	199809	89314623	21.1424	7.6460
393	154449	60698457	19.8242	7.3248	448	200704	89915392	21.1660	7.6517
394	155236	61162984	19.8494	7.3310	449	201601	90518849	21.1896	7.6574
395	156025	61629875	19.8746	7.3372					
396	156816	62099136	19.8997	7.3434	450	202500	91125000	21.2132	7.6631
397	157609	62570773	19.9249	7.3496	451	203401	91733851	21.2368	7.6688
398	158404	63044792	19.9499	7.3558	452	204304	92345408	21.2603	7.6744
399	159201	63521199	19.9750	7.3619	453	205209	92959677	21.2838	7.6801
					454	206116	93576664	21.3073	7.6857
400	160000	64000000	20.0000	7.3681	455	207025	94196375	21.3307	7.6914
401	160801	64481201	20.0250	7.3742	456	207936	94818816	21.3542	7.6970
402	161604	64964808	20.0499	7.3803	457	208849	95443993	21.3776	7.7026
403	162409	65450827	20.0749	7.3864	458	209764	96071912	21.4009	7.7082
404	163216	65939264	20.0998	7.3925	459	210681	96702579	21.4243	7.7138
405	164025	66430125	20.1246	7.3986					
406	164836	66923416	20.1494	7.4047	460	211600	97336000	21.4476	7.7194
407	165649	67419143	20.1742	7.4108	461	212521	97972181	21.4709	7.7250
408	166464	67917312	20.1990	7.4169	462	213444	98611128	21.4942	7.7306
409	167281	68417929	20.2237	7.4229	463	214369	99252847	21.5174	7.7362
					464	215296	99897344	21.5407	7.7418
410	168100	68921000	20.2485	7.4290	465	216225	100544625	21.5639	7.7473
411	168921	69426531	20.2731	7.4350	466	217156	101194696	21.5870	7.7529
412	169744	69934528	20.2978	7.4410	467	218089	101847563	21.6102	7.7584
413	170569	70444997	20.3224	7.4470	468	219024	102503232	21.6333	7.7639
414	171396	70957944	20.3470	7.4530	469	219961	103161709	21.6564	7.7695
415	172225	71473375	20.3715	7.4590					

No.	Square	Cube	Square Root	Cube Root
470	220900	103823000	21.6795	7.7750
471	221841	104487111	21.7025	7.7805
472	222784	105154048	21.7256	7.7860
473	223729	105823817	21.7486	7.7915
474	224676	106496424	21.7715	7.7970
475	225625	107171875	21.7945	7.8025
476	226576	107850176	21.8174	7.8079
477	227529	108531333	21.8403	7.8134
478	228484	109215352	21.8632	7.8188
479	229441	109902239	21.8861	7.8243
480	230400	110592000	21.9089	7.8297
481	231361	111284641	21.9317	7.8352
482	232324	111980168	21.9545	7.8406
483	233289	112678587	21.9773	7.8460
484	234256	113379904	22.0000	7.8514
485	235225	114084125	22.0227	7.8568
486	236196	114791256	22.0454	7.8622
487	237169	115501303	22.0681	7.8676
488	238144	116214272	22.0907	7.8730
489	239121	116930169	22.1133	7.8784
490	240100	117649000	22.1359	7.8837
491	241081	118370771	22.1585	7.8891
492	242064	119095488	22.1811	7.8944
493	243049	119823157	22.2036	7.8998
494	244036	120553784	22.2261	7.9051
495	245025	121287375	22.2486	7.9105
496	246016	122023936	22.2711	7.9158
497	247009	122763473	22.2935	7.9211
498	248004	123505992	22.3159	7.9264
499	249001	124251499	22.3383	7.9317
500	250000	125000000	22.3607	7.9370
501	251001	125751501	22.3830	7.9423
502	252004	126506008	22.4054	7.9476
503	253009	127263527	22.4277	7.9528
504	254016	128024064	22.4499	7.9581
505	255025	128787625	22.4722	7.9634
506	256036	129554216	22.4944	7.9686
507	257049	130323843	22.5167	7.9739
508	258064	131096512	22.5389	7.9791
509	259081	131872229	22.5610	7.9843
510	260100	132651000	22.5832	7.9896
511	261121	133432831	22.6053	7.9948
512	262144	134217728	22.6274	8.0000
513	263169	135005697	22.6495	8.0052
514	264196	135796744	22.6716	8.0104
515	265225	136590875	22.6936	8.0156
516	266256	137388096	22.7156	8.0208
517	267289	138188413	22.7376	8.0260
518	268324	138991832	22.7596	8.0311
519	269361	139798359	22.7816	8.0363
520	270400	140608000	22.8035	8.0415
521	271441	141420761	22.8254	8.0466
522	272484	142236648	22.8473	8.0517
523	273529	143055667	22.8692	8.0569

No.	Square	Cube	Square Root	Cube Root
524	274576	143877824	22.8910	8.0620
525	275625	144703125	22.9129	8.0671
526	276676	145531576	22.9347	8.0723
527	277729	146363183	22.9565	8.0774
528	278784	147197952	22.9783	8.0825
529	279841	148035889	23.0000	8.0876
530	280900	148877000	23.0217	8.0927
531	281961	149721291	23.0434	8.0978
532	283024	150568768	23.0651	8.1028
533	284089	151419437	23.0868	8.1079
534	285156	152273304	23.1084	8.1130
535	286225	153130375	23.1301	8.1180
536	287296	153990656	23.1517	8.1231
537	288369	154854153	23.1733	8.1281
538	289444	155720872	23.1948	8.1332
539	290521	156590819	23.2164	8.1382
540	291600	157464000	23.2379	8.1433
541	292681	158340421	23.2594	8.1483
542	293764	159220088	23.2809	8.1533
543	294849	160103007	23.3024	8.1583
544	295936	160989184	23.3238	8.1633
545	297025	161878625	23.3452	8.1683
546	298116	162771336	23.3666	8.1733
547	299209	163667323	23.3880	8.1783
548	300304	164566592	23.4094	8.1833
549	301401	165469149	23.4307	8.1882
550	302500	166375000	23.4521	8.1932
551	303601	167284151	23.4734	8.1982
552	304704	168196608	23.4947	8.2031
553	305809	169112377	23.5160	8.2081
554	306916	170031464	23.5372	8.2130
555	308025	170953875	23.5584	8.2180
556	309136	171879616	23.5797	8.2229
557	310249	172808693	23.6008	8.2278
558	311364	173741112	23.6220	8.2327
559	312481	174676879	23.6432	8.2377
560	313600	175616000	23.6643	8.2426
561	314721	175558481	23.6854	8.2475
562	315844	177504328	23.7065	8.2524
563	316969	178453547	23.7276	8.2573
564	318096	179406144	23.7487	8.2621
565	319225	180362125	23.7697	8.2670
566	320356	181321496	23.7908	8.2719
567	321489	182284263	23.8118	8.2768
568	322624	183250432	23.8328	8.2816
569	323761	184220009	23.8537	8.2865
570	324900	185193000	23.8747	8.2913
571	326041	186169411	23.8956	8.2962
572	327184	187149248	23.9165	8.3010
573	328329	188132517	23.9374	8.3059
574	329476	189119224	23.9583	8.3107
575	330625	190109375	23.9792	8.3155
576	331776	191102976	24.0000	8.3203
577	332929	192100033	24.0208	8.3251
578	334084	193100552	24.0416	8.3300
579	335241	194104539	24.0624	8.3348

No.	Square	Cube	Square Root	Cube Root
580	336400	195112000	24.0832	8.3396
581	337561	196122941	24.1039	8.3443
582	338724	197137368	24.1247	8.3491
583	339889	198155287	24.1454	8.3539
584	341056	199176704	24.1661	8.3587
585	342225	200201625	24.1868	8.3634
586	343396	201230056	24.2074	8.3682
587	344569	202262003	24.2281	8.3730
588	345744	203297472	24.2487	8.3777
589	346921	204336469	24.2693	8.3825
590	348100	205379000	24.2899	8.3872
591	349281	206425071	24.3105	8.3919
592	350464	207474688	24.3311	8.3967
593	351649	208527857	24.3516	8.4014
594	352836	209584584	24.3721	8.4061
595	354025	210644875	24.3926	8.4108
596	355216	211708736	24.4131	8.4155
597	356409	212776173	24.4336	8.4202
598	357604	213847192	24.4540	8.4249
599	358801	214921799	24.4745	8.4296
600	360000	216000000	24.4949	8.4343
601	361201	217081801	24.5153	8.4390
602	362404	218167208	24.5357	8.4437
603	363609	219256227	24.5561	8.4484
604	364816	220348864	24.5764	8.4530
605	366025	221445125	24.5967	8.4577
606	367236	222545016	24.6171	8.4623
607	368449	223648543	24.6374	8.4670
608	369664	224755712	24.6577	8.4716
609	370881	225866529	24.6779	8.4763
610	372100	226981000	24.6982	8.4809
611	373321	228099131	24.7184	8.4856
612	374544	229220928	24.7386	8.4902
613	375769	230346397	24.7588	8.4948
614	376996	231475544	24.7790	8.4994
615	378225	232608375	24.7992	8.5040
616	379456	233744896	24.8193	8.5086
617	380689	234885113	24.8395	8.5132
618	381924	236029032	24.8596	8.5178
619	383161	237176659	24.8797	8.5224
620	384400	238328000	24.8998	8.5270
621	385641	239483061	24.9199	8.5316
622	386884	240641848	24.9399	8.5462
623	388129	241804367	24.9600	8.5408
624	389376	242970624	24.9800	8.5453
625	390625	244140625	25.0000	8.5499
626	391876	245314376	25.0200	8.5544
627	393129	246491883	25.0400	8.5590
628	394384	247673152	25.0599	8.5635
629	395641	248858189	25.0799	8.5681
630	396900	250047000	25.0998	8.5726
631	398161	251239591	25.1197	8.5772
632	399424	252435968	25.1396	8.5817
633	400689	253636137	25.1595	8.5862
634	401956	254840104	25.1794	8.5907

No.	Square	Cube	Square Root	Cube Root
635	403225	256047875	25.1992	8.5952
636	404496	257259456	25.2190	8.5997
637	405769	258474853	25.2389	8.6043
638	407044	259694072	24.2587	8.6088
639	408321	260917119	25.2784	8.6132
640	409600	262144000	25.2982	8.6177
641	410881	263374721	25.3180	8.6222
642	412164	264609288	25.3377	8.6267
643	413449	265847707	25.3574	8.6312
644	414736	267089984	25.3772	8.6357
645	416025	268336125	25.3969	8.6401
646	417316	269586136	25.4165	8.6446
647	418609	270840023	25.4362	8.6490
648	419904	272097792	25.4558	8.6535
649	421201	273359449	25.4755	8.6579
650	422500	274625000	25.4951	8.6624
651	423801	275894451	25.5147	8.6668
652	425104	277167808	25.5343	8.6713
653	426409	278445077	25.5539	8.6757
654	427716	279726264	25.5734	8.6801
655	429025	281011375	25.5930	8.6845
656	430336	282300416	25.6125	8.6890
657	431649	283593393	25.6320	8.6934
658	432964	284890312	25.6515	8.6978
659	434281	286191179	25.6710	8.7022
660	435600	287496000	25.6905	8.7066
661	436921	288804781	25.7099	8.7110
662	438244	290117528	25.7294	8.7154
663	439569	291434247	25.7488	8.7198
664	440896	292754944	25.7682	8.7241
665	442225	294079625	25.7876	8.7285
666	443556	295408296	25.8070	8.7329
667	444889	296740963	25.8263	8.7373
668	446224	298077632	25.8457	8.7416
669	447561	299418309	25.8650	8.7460
670	448900	300763000	25.8844	8.7503
671	450241	302111711	25.9037	8.7547
672	451584	303464448	25.9230	8.7590
673	452929	304821217	25.9422	8.7634
674	454276	306182024	25.9615	8.7677
675	455625	307546875	25.9808	8.7721
676	456976	308915776	26.0000	8.7764
677	458329	310288733	26.0192	8.7807
678	459684	311665752	26.0384	8.7850
679	461041	313046839	26.0576	8.7893
680	462400	314432000	26.0768	8.7937
681	463761	315821241	26.0960	8.7980
682	465124	317214568	26.1151	8.8023
683	466489	318611987	26.1343	8.8066
684	467856	320013504	26.1534	8.8109
685	469225	321419125	26.1725	8.8152
686	470596	322828856	26.1916	8.8194
687	471969	324242703	26.2107	8.8237
688	473344	325660672	26.2298	8.8280
689	474721	327082769	26.2488	8.8323

No.	Square	Cube	Square Root	Cube Root
690	476100	328509000	26.2679	8.8366
691	477481	329939371	26.2869	8.8408
692	478864	331373888	26.3059	8.8451
693	480249	332812557	26.3249	8.8493
694	481636	334255384	26.3439	8.8536
695	483025	335702375	26.3629	8.8578
696	484416	337153536	26.3818	8.8621
697	485809	338608873	26.4008	8.8663
698	487204	340068392	26.4197	8.8706
699	488601	341532099	26.4386	8.8748
700	490000	343000000	26.4575	8.8790
701	491401	344472101	26.4764	8.8833
702	492804	345948408	26.4953	8.8875
703	494209	347428927	26.5141	8.8917
704	495616	348913664	26.5330	8.8959
705	497025	350402625	26.5518	8.9001
706	498436	351895816	26.5707	8.9043
707	499849	353393243	26.5895	8.9085
708	501264	354894912	26.6083	8.9127
709	502681	356400829	26.6271	8.9169
710	504100	357911000	26.6458	8.9211
711	505521	359425431	26.6646	8.9253
712	506944	360944128	26.6833	8.9295
713	508369	362467097	26.7021	8.9337
714	509796	363994344	26.7208	8.9378
715	511225	365525875	26.7395	8.9420
716	512656	367061696	26.7582	8.9462
717	514089	368601813	26.7769	8.9503
718	515524	370146232	26.7955	8.9545
719	516961	371694959	26.8142	8.9587
720	518400	373248000	26.8328	8.9628
721	519841	374805361	26.8514	8.9670
722	521284	376367048	26.8701	8.9711
723	522729	377933067	26.8887	8.9752
724	524176	379503424	26.9072	8.9794
725	525625	381078125	26.9258	8.9835
726	527076	382657176	26.9444	8.9876
727	528529	384240583	26.9629	8.9918
728	529984	385828352	26.9815	8.9959
729	531441	387420489	27.0000	9.0000
730	532900	389017000	27.0185	9.0041
731	534361	390617891	27.0370	9.0082
732	535824	392223168	27.0555	9.0123
733	537289	393832837	27.0740	9.0164
734	538756	395446904	27.0924	9.0205
735	540225	397065375	27.1109	9.0246
736	541696	398688256	27.1293	9.0287
737	543169	400315553	27.1477	9.0328
738	544644	401947272	27.1662	9.0369
739	546121	403583419	27.1846	9.0410
740	547600	405224000	27.2029	9.0450
741	549081	406869021	27.2213	9.0491
742	550564	408518488	27.2397	9.0532
743	552049	410172407	27.2580	9.0572
744	553536	411830784	27.2764	9.0613
745	555025	413493625	27.2947	9.0654
746	556516	415160936	27.3130	9.0694
747	558009	416832723	27.3313	9.0735
748	559504	418508992	27.3496	9.0775
749	561001	420189749	27.3679	9.0816
750	562500	421875000	27.3861	9.0856
751	564001	423564751	27.4044	9.0896
752	565504	425259008	27.4226	9.0937
753	567009	426957777	27.4408	9.0977
754	568516	428661064	27.4591	9.1017
755	570025	430368875	27.4773	9.1057
756	571536	432081216	27.4955	9.1098
757	573049	433798093	27.5136	9.1138
758	574564	435519512	27.5318	9.1178
759	576081	437245479	27.5500	9.1218
760	577600	438976000	27.5681	9.1258
761	579121	440711081	27.5862	9.1298
762	580644	442450728	27.6043	9.1338
763	582169	444194947	27.6225	9.1378
764	583696	445943744	27.6405	9.1418
765	585225	447697125	27.6586	9.1458
766	586756	449455096	27.6767	9.1498
767	588289	451217663	27.6948	9.1537
768	589824	452984832	27.7128	9.1577
769	591361	454756609	27.7308	9.1617
770	592900	456533000	27.7489	9.1657
771	594441	458314011	27.7669	9.1696
772	595984	460099648	27.7849	9.1736
773	597529	461889917	27.8029	9.1775
774	599076	463684824	27.8209	9.1815
775	600625	465484375	27.8388	9.1855
776	602176	467288576	27.8568	9.1894
777	603729	469097433	27.8747	9.1933
778	605284	470910952	27.8927	9.1973
779	606841	472729139	27.9106	9.2012
780	608400	474552000	27.9285	9.2052
781	609961	476379541	27.9464	9.2091
782	611524	478211768	27.9643	9.2130
783	613089	480048687	27.9821	9.2170
784	614656	481890304	28.0000	9.2209
785	616225	483736625	28.0179	9.2248
786	617796	485587656	28.0357	9.2287
787	619369	487443403	28.0535	9.2326
788	620944	489303872	28.0713	9.2365
789	622521	491169069	28.0891	9.2404
790	624100	493039000	28.1069	9.2443
791	625681	494913671	28.1247	9.2482
792	627264	496793088	28.1425	9.2521
793	628849	498677257	28.1603	9.2560
794	630436	500566184	28.1780	9.2599
795	632025	502459875	28.1957	9.2638
796	633616	504358336	28.2135	9.2677
797	635209	506261573	28.2312	9.2716
798	636804	508169592	28.2489	9.2754
799	638401	510082399	28.2666	9.2793

No.	Square	Cube	Square Root	Cube Root	No.	Square	Cube	Square Root	Cube Root
800	640000	512000000	28.2843	9.2832	855	731025	625026375	29.2404	9.4912
801	641601	513922401	28.3019	9.2870	856	732736	627222016	29.2575	9.4949
802	643204	515849608	28.3196	9.2909	857	734449	629422793	29.2746	9.4986
803	644809	517781627	28.3373	9.2948	858	736164	631628712	29.2916	9.5023
804	646416	519718464	28.3549	9.2986	859	737881	633839779	-29.3087	9.5060
805	648025	521660125	28.3725	9.3025					
806	649636	523606616	28.3901	9.3063	860	739600	636056000	29.3258	9.5097
807	651249	525557943	28.4077	9.3102	861	741321	638277381	29.3428	9.5134
808	652864	527514112	28.4253	9.3140	862	743044	640503928	29.3598	9.5171
809	654481	529475129	28.4429	9.3179	863	744769	642735647	29.3769	9.5207
					864	746496	644972544	29.3939	9.5244
810	656100	531441000	28.4605	9.3217	865	748225	647214625	29.4109	9.5281
811	657721	533411731	28.4781	9.3255	866	749956	649461896	29.4279	9.5317
812	659344	535387328	28.4956	9.3294	867	751689	651714363	29.4449	9.5354
813	660969	537367797	28.5132	9.3332	868	753424	653972032	29.4618	9.5391
814	662596	539353144	28.5307	9.3370	869	755161	656234909	29.4788	9.5427
815	664225	541343375	28.5482	9.3408					
816	665856	543338496	28.5657	9.3447	870	756900	658503000	29.4958	9.5464
817	667489	545338513	28.5832	9.3485	871	758641	660776311	29.5127	9.5501
818	669124	547343432	28.6007	9.3523	872	760384	663054848	29.5296	9.5537
819	670761	549353259	28.6182	9.3561	873	762129	665338617	29.5466	9.5574
					874	763876	667627624	29.5635	9.5610
820	672400	551368000	28.6356	9.3599	875	765625	669921875	29.5804	9.5647
821	674041	553387661	28.6531	9.3637	876	767376	672221376	29.5973	9.5683
822	675684	555412248	28.6705	9.3675	877	769129	574526133	29.6142	9.5719
823	677329	557441767	28.6880	9.3713	878	770884	676836152	29.6311	9.5756
824	678976	559476224	28.7054	9.3751	879	772641	679151439	29.6479	9.5792
825	680625	561515625	28.7228	9.3789					
826	682276	563559976	28.7402	9.3827	880	774400	681472000	29.6648	9.5828
827	683929	565609283	28.7576	9.3865	881	776161	683797841	29.6816	9.5865
828	685584	567663552	28.7750	9.3902	882	777924	686128968	29.6985	9.5901
829	687241	569722789	28.7924	9.3940	883	779689	688465387	29.7153	9.5937
					884	781456	690807104	29.7321	9.5973
830	688900	571787000	28.8097	9.3978	885	783225	693154125	29.7489	9.6010
831	690561	573856191	28.8271	9.4016	886	784996	695506456	29.7658	9.6046
832	692224	575930368	28.8444	9.4053	887	786769	697864103	29.7825	9.6082
833	693889	578009537	28.8617	9.4091	888	788544	700227072	29.7993	9.6118
834	695556	580093704	28.8791	9.4129	889	790321	702595369	29.8161	9.6154
835	697225	582182875	28.8964	9.4166					
836	698896	584277056	28.9137	9.4204	890	792100	704969000	29.8329	9.6190
837	700569	586376253	28.9310	9.4241	891	793881	707347971	29.8496	9.6226
838	702244	588480472	28.9482	9.4279	892	795664	709732288	29.8664	9.6262
839	703921	590589719	28.9655	9.4316	893	797449	712121957	29.8831	9.6298
					894	799236	714516984	29.8998	9.6334
840	705600	592704000	28.9828	9.4354	895	801025	716917375	29.9166	9.6370
841	707281	594823321	29.0000	9.4391	896	802816	719323136	29.9333	9.6406
842	708964	596947688	29.0172	9.4429	897	804609	721734273	29.9500	9.6442
843	710649	599077107	29.0345	9.4466	898	806404	724150792	29.9666	9.6477
844	712336	601211584	29.0517	9.4503	899	808201	726572699	29.9833	9.6513
845	714025	603351125	29.0689	9.4541					
846	715716	605495736	29.0861	9.4578	900	810000	729000000	30.0000	9.6549
847	717409	607645423	29.1033	9.4615	901	811801	731432701	30.0167	9.6585
848	719104	609800192	29.1204	9.4652	902	813604	733870808	30.0333	9.6620
849	720801	611960049	29.1376	9.4690	903	815409	736314327	30.0500	9.6656
					904	817216	738763264	30.0666	9.6692
850	722500	614125000	29.1548	9.4727	905	819025	741217625	30.0832	9.6727
851	724201	616295051	29.1719	9.4764	906	820836	743677416	30.0998	9.6763
852	725904	618470208	29.1890	9.4801	907	822649	746142643	30.1164	9.6799
853	727609	620650477	29.2062	9.4838	908	824464	748613312	30.1330	9.6834
854	729316	622835864	29.2233	9.4875	909	826281	751089429	30.1496	9.6870

No.	Square	Cube	Square Root	Cube Root	No.	Square	Cube	Square Root	Cube Root
					955	912025	870983875	30.9031	9.8477
910	828100	753571000	30.1662	9.6905	956	913936	873722816	30.9192	9.8511
911	829921	756058031	30.1828	9.6941	957	915849	876467493	30.9354	9.8546
912	831744	758550528	30.1993	9.6976	958	917764	879217912	30.9516	9.8580
913	833569	761048497	30.2159	9.7012	959	919681	881974079	30.9677	9.8614
914	835396	763551944	30.2324	9.7017					
915	837225	766060875	30.2490	9.7082	960	921600	884736000	30.9839	9.8648
916	839056	768575296	30.2655	9.7118	961	923521	887503681	31.0000	9.8683
917	840889	771095213	30.2820	9.7153	962	925444	890277128	31.0161	9.8717
918	842724	773620632	30.2985	9.7188	963	927369	893056347	31.0322	9.8751
919	844561	776151559	30.3150	9.7224	964	929296	895841344	31.0483	9.8785
					965	931225	898632125	31.0644	9.8819
920	846400	778688000	30.3315	9.7259	966	933156	901428696	31.0805	9.8854
921	848241	781229961	30.3480	9.7294	967	935089	904231063	31.0966	9.8888
922	850084	783777448	30.3645	9.7329	968	937024	907039232	31.1127	9.8922
923	851929	786330467	30.3809	9.7364	969	938961	909853209	31.1288	9.8956
924	853776	788889024	30.3974	9.7400					
925	855625	791453125	30.4138	9.7435	970	940900	912673000	31.1448	9.8990
926	857476	794022776	30.4302	9.7470	971	942841	915498611	31.1609	9.9024
927	859329	796597983	30.4467	9.7505	972	944784	918330048	31.1769	9.9058
928	861184	799178752	30.4631	9.7540	973	946729	921167317	31.1929	9.9092
929	863041	801765089	30.4795	9.7575	974	948676	924010424	31.2090	9.9126
					975	950625	926859375	31.2250	9.9160
930	864900	804357000	30.4959	9.7610	976	952576	929714176	31.2410	9.9194
931	866761	806954491	30.5123	9.7645	977	954529	932574833	31.2570	9.9227
932	868624	809557568	30.5287	9.7680	978	956484	935441352	31.2730	9.9261
933	870489	812166237	30.5450	9.7715	979	958441	938313739	31.2890	9.9295
934	872356	814780504	30.5614	9.7750					
935	874225	817400375	30.5778	9.7785	980	960400	941192000	31.3050	9.9329
936	876096	820025856	30.5941	9.7819	981	962361	944076141	31.3209	9.9363
937	877969	822656953	30.6105	9.7854	982	964324	946966168	31.3369	9.9396
938	879844	825293672	30.6268	9.7889	983	966289	949862087	31.3528	9.9430
939	881721	827936019	30.6431	9.7924	984	968256	952763904	31.3688	9.9464
					985	970225	955671625	31.3847	9.9497
940	883600	830584000	30.6594	9.7959	986	972196	958585256	31.4006	9.9531
941	885481	833237621	30.6757	9.7993	987	974169	961504803	31.4166	9.9565
942	887364	835896888	30.6920	9.8028	988	976144	964430272	31.4325	9.9598
943	889249	838561807	30.7083	9.8063	989	978121	967361669	31.4484	9.9632
944	891136	841232384	30.7246	9.8097					
945	893025	843908625	30.7409	9.8132	990	980100	970299000	31.4643	9.9666
946	894916	846590536	30.7571	9.8167	991	982081	973242271	31.4802	9.9699
947	896809	849278123	30.7734	9.8201	992	984064	976191488	31.4960	9.9733
948	898704	851971392	30.7896	9.8236	993	986049	979146657	31.5119	9.9766
949	900601	854670349	30.8058	9.8270	994	988036	982107784	31.5278	9.9800
					995	990025	985074875	31.5436	9.9833
950	902500	857375000	30.8221	9.8305	996	992016	988047936	31.5595	9.9866
951	904401	860085351	30.8383	9.8339	997	994009	991026973	31.5753	9.9900
952	906304	862801408	30.8545	9.8374	998	996004	994011992	31.5911	9.9933
953	908209	865523177	30.8707	9.8408	999	998001	997002999	31.6070	9.9967
954	910116	868250664	30.8869	9.8443					

Appendix IV

PREFIXES AND SYMBOLS FOR POWERS OF TEN

Multiples and Submultiples (Powers of Ten)	Prefixes	Symbols
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^4	myria	Ma
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p (formerly micromicro $-\mu\mu$)
10^{-15}	femto	f
10^{-18}	atto	a
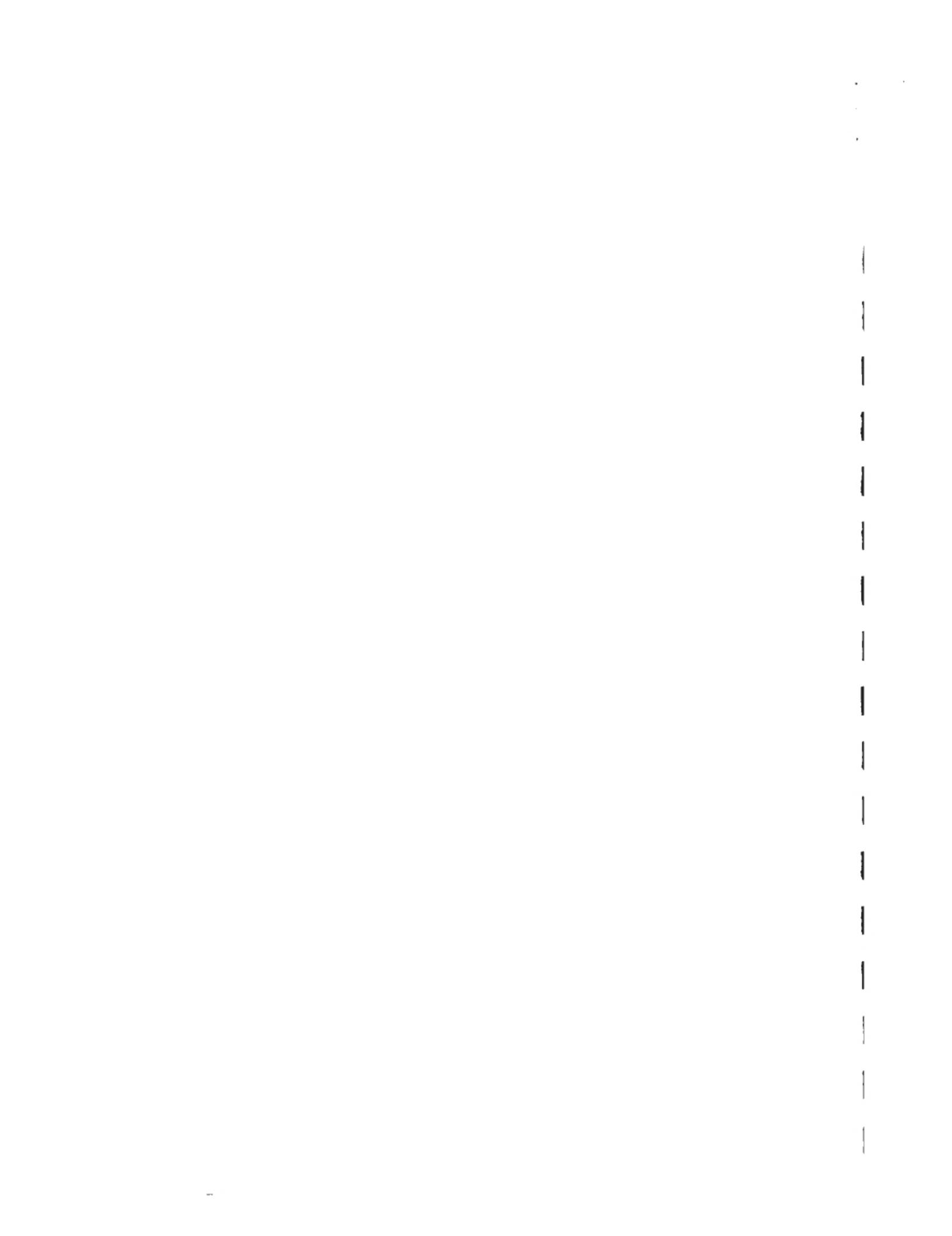
PERIODIC TABLE OF THE ELEMENTS

ATOMIC NUMBER — 1
ELEMENT SYMBOL — H
ATOMIC WEIGHT — 1.00797

LIGHT METALS — HEAVY METALS — NONMETALS — INERT GASES

IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIII	VIII	VIII	IB	IIB	IIIA	IVA	VA	VIA	VIIA	Inert Gases
1 H 1.00797																	2 He 4.0026
3 Li 6.939	4 Be 9.0122											5 B 10.811	6 C 12.0115	7 N 14.0067	8 O 15.9994	9 F 18.9984	10 Ne 20.183
11 Na 22.9898	12 Mg 24.312											13 Al 26.98	14 Si 28.086	15 P 30.9738	16 S 32.064	17 Cl 35.453	18 A 39.948
19 K 39.102	20 Ca 40.08	21 Sc 44.956	22 Ti 47.90	23 V 50.942	24 Cr 51.996	25 Mn 54.9380	26 Fe 55.847	27 Co 58.9332	28 Ni 58.71	29 Cu 63.54	30 Zn 65.37	31 Ga 69.72	32 Ge 72.59	33 As 74.9216	34 Se 78.96	35 Br 79.909	36 Kr 83.80
37 Rb 85.47	38 Sr 87.62	39 Y 88.905	40 Zr 91.22	41 Cb 92.906	42 Mo 95.94	43 Tc (99)	44 Ru 101.07	45 Rh 102.905	46 Pd 106.4	47 Ag 107.870	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.9044	54 Xe 131.30
55 Cs 132.905	56 Ba 137.34	57 La 138.91	72 Hf 178.49	73 Ta 180.948	74 W 183.85	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.09	79 Au 196.967	80 Hg 200.59	81 Tl 204.37	82 Pb 207.19	83 Bi 208.980	84 Po (210)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	89 Ac (227)															

LANTHANUM SERIES

58 Ce 140.12	59 Pr 140.907	60 Nd 144.24	61 Pm (147)	62 Sm 150.35	63 Eu 151.96	64 Gd 157.25	65 Tb 158.924	66 Dy 162.50	67 Ho 164.930	68 Er 167.26	69 Tm 168.934	70 Yb 173.04	71 Lu 174.97

ACTINIUM SERIES

90 Th 232.038	91 Pa (230)	92 U 238.03	93 Np (237)	94 Pu (242)	95 Am (243)	96 Cm (247)	97 Bk (249)	98 Cf (251)	99 E (254)	100 Fm (253)	101 Md (256)	102 (254)	103 Lw (257)	104 (258-261)	105 (258-261)

● — INDICATES PRINCIPAL RADIOACTIVE ELEMENTS

Symbol	Name	Atomic Number	Atomic Weight
A	Argon	18	39. 948
Ac	Actinium	89	(227)
Ag	Silver	47	107. 870
Al	Aluminum	13	26. 98
Am	Americum	95	(243)
As	Arsenic	33	74. 9216
At	Astatine	85	(210)
Au	Gold	79	196. 967
B	Boron	5	10. 811
Ba	Barium	56	137. 34
Be	Beryllium	4	9. 0122
Bi	Bismuth	83	208. 980
Bk	Berkelium	97	(249)
Br	Bromine	35	79. 909
C	Carbon	6	12. 01115
Ca	Calcium	20	40. 08
Cb	Columbium (Niobium)	41	92. 906
Cd	Cadmium	48	112. 40
Ce	Cerium	58	140. 12
Cf	Californium	98	(251)
Cl	Chlorine	17	35. 453
Cm	Curium	96	(247)
Co	Cobalt	27	58. 9332
Cr	Chromium	24	51. 996
Cs	Cesium	55	132. 905
Cu	Copper	29	63. 54
Dy	Dysprosium	66	162. 50
E	Einsteinium	99	(254)
Er	Erbium	68	167. 26
Eu	Europium	63	151. 96
F	Fluorine	9	18. 9984
Fe	Iron	26	55. 847
Fm	Fermium	100	(253)
Fr	Francium	87	(223)
Ga	Gallium	31	69. 72
Gd	Gadolium	64	157. 25
Ge	Germanium	32	72. 59
H	Hydrogen	1	1. 00797
He	Helium	2	4. 0026
Hf	Hafnium	72	178. 49
Hg	Mercury	80	200. 59
Ho	Holmium	67	164. 930
I	Iodine	53	126. 9044
In	Indium	49	114. 82
Ir	Iridium	77	192. 2
K	Potassium	19	39. 102
Kr	Krypton	36	83. 80
La	Lanthanum	57	138. 91
Li	Lithium	3	6. 939
Lu	Lutetium	71	174. 97
Lw	Lawrencium	103	(257)
Md	Mendelevium	101	(256)
Mg	Magnesium	12	24. 312
Mn	Manganese	25	54. 9380
Mo	Molybdenum	42	95. 94
N	Nitrogen	7	14. 0067
Na	Sodium	11	22. 9898

Symbol	Name	Atomic Number	Atomic Weight
Nd	Neodymium	60	144. 24
Ne	Neon	10	20. 183
Ni	Nickel	28	58. 71
* No	Nobelium	102	(254)
Np	Neptunium	93	(237)
O	Oxygen	8	15. 9994
Os	Osmium	76	190. 2
P	Phosphorus	15	30. 9738
Pa	Protoactinium	91	(231)
Pb	Lead	82	207. 19
Pd	Palladium	46	106. 4
Pm	Promethium	61	(147)
Po	Polonium	84	(210)
Pr	Praseodymium	59	140. 907
Pt	Platinum	78	195. 09
Pu	Plutonium	94	(242)
Ra	Radium	88	(226)
Rb	Rubidium	37	85. 47
Re	Rhenium	75	186. 2
Rh	Rhodium	45	102. 905
Rn	Radon	86	(222)
Ru	Ruthenium	44	101. 07
S	Sulfur	16	32. 064
Sb	Antimony	51	121. 75
Sc	Scandium	21	44. 956
Se	Selenium	34	78. 96
Si	Silicon	14	28. 086
Sm	Samarium	62	150. 35
Sn	Tin	50	118. 69
Sr	Strontium	38	87. 62
Ta	Tantalum	73	180. 948
Tb	Terbium	65	158. 924
Tc	Technetium	43	(99)
Te	Tellurium	52	127. 60
Th	Thorium	90	232. 038
Ti	Titanium	22	47. 90
Tl	Thallium	81	204. 37
Tm	Thulium	69	168. 934
U	Uranium	92	238. 03
V	Vanadium	23	50. 942
W	Tungsten	74	183. 85
Xe	Xenon	54	131. 30
Y	Yttrium	39	88. 905
Yb	Ytterbium	70	173. 04
Zn	Zinc	30	65. 37
Zr	Zirionium	40	91. 22

* Note: Element proposed but not confirmed.

Name	Symbol	Atomic Number	Atomic Weight
Actinium	Ac	89	(227)
Aluminum	Al	13	26.98
Americum	Am	95	(243)
Antimony	Sb	51	121.75
Argon	A	18	39.948
Arsenic	As	33	74.9216
Astatine	At	85	(210)
Barium	Ba	56	137.34
Berkelium	Bk	97	(249)
Beryllium	Be	4	9.0122
Bismuth	Bi	83	208.980
Boron	B	5	10.811
Bromine	Br	35	79.909
Cadmium	Cd	48	112.40
Calcium	Ca	20	40.08
Californium	Cf	98	(251)
Carbon	C	6	12.01115
Cerium	Ce	58	140.12
Cesium	Cs	55	132.905
Chlorine	Cl	17	35.453
Chromium	Cr	24	51.996
Cobalt	Co	27	58.9332
Columbium (Niobium)	Cb	41	92.906
Copper	Cu	29	63.54
Curium	Cm	96	(247)
Dysprosium	Dy	66	162.50
Einsteinium	E	99	(254)
Erbium	Er	68	167.26
Europium	Eu	63	151.96
Fermium	Fm	100	(253)
Fluorine	F	9	18.9984
Francium	Fr	87	(223)
Gadolium	Gd	64	157.25
Gallium	Ga	31	69.72
Germanium	Ge	32	72.59
Gold	Au	79	196.967
Hafnium	Hf	72	178.49
Helium	He	2	4.0026
Holmium	Ho	67	164.930
Hydrogen	H	1	1.00797
Indium	In	49	114.82
Iodine	I	53	126.9044
Iridium	Ir	77	192.2
Iron	Fe	26	55.847
Krypton	Kr	36	83.80
Lanthanum	La	57	138.91
Lawrencium	Lw	103	(257)
Lead	Pb	82	207.19
Lithium	Li	3	6.939
Lutetium	Lu	71	174.97
Magnesium	Mg	12	24.312
Manganese	Mn	25	54.9380
Mercury	Hg	80	200.59
Mendelevium	Md	101	(256)
Molybdenum	Mo	42	95.94
Neodymium	Nd	60	144.24
Neon	Ne	10	20.183

Name	Symbol	Atomic Number	Atomic Weight
Neptunium	Np	93	(237)
Nickel	Ni	28	58. 71
Niobium (See Columbium)			
Nitrogen	N	7	14. 0067
* Nobelium	No	102	(254)
Osmium	Os	76	190. 2
Oxygen	O	8	15. 9994
Palladium	Pd	46	106. 4
Phosphorus	P	15	30. 9738
Platinum	Pt	78	195. 09
Plutonium	Pu	94	(242)
Polonium	Po	84	(210)
Potassium	K	19	39. 102
Praseodymium	Pr	59	140. 907
Promethium	Pm	61	(147)
Protoactinium	Pa	91	(231)
Radium	Ra	88	(226)
Radon	Rn	86	(222)
Rhenium	Re	75	186. 2
Rhodium	Rh	45	102. 905
Rubidium	Rb	37	85. 47
Ruthenium	Ru	44	101. 07
Samarium	Sm	62	150. 35
Scandium	Sc	21	44. 956
Selenium	Se	34	78. 96
Silicon	Si	14	28. 086
Silver	Ag	47	107. 870
Sodium	Na	11	22. 9898
Strontium	Sr	38	87. 62
Sulfur	S	16	32. 064
Tantalum	Ta	73	180. 948
Technetium	Tc	43	(99)
Tellurium	Te	52	127. 60
Terbium	Tb	65	158. 924
Thallium	Tl	81	204. 37
Thorium	Th	90	232. 038
Thulium	Tm	69	168. 934
Tin	Sn	50	118. 69
Titanium	Ti	22	47. 90
Tungsten	W	74	183. 85
Uranium	U	92	238. 03
Vanadium	V	23	50. 942
Xenon	Xe	54	131. 30
Ytterbium	Yb	70	173. 04
Yttrium	Y	39	88. 905
Zinc	Zn	30	65. 37
Zirconium	Zr	40	91. 22

* Note: Element proposed but not confirmed.

Appendix VI

ELECTRONIC COLOR CODES

The various types of capacitors are coded in a manner somewhat similar to the method used to color-code resistors. There are two color-coding systems for capacitors. One is the JAN (Joint Army Navy) system by which all capacitors produced for military use are marked. The other is RMA (Radio Manufacturers Association) system—recently renamed the EIA (Electronic Industries Association) system.
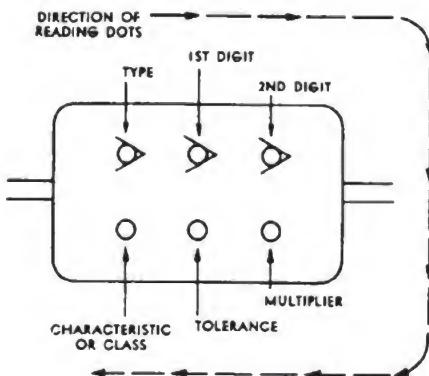
TYPE	COLOR	1ST DIGIT	2ND DIGIT	MULTIPLIER	TOLERANCE (PERCENT)	CHARACTERISTIC OR CLASS
JAN, MICA	BLACK	0	0	1.0		APPLIES TO
	BROWN	1	1	10	± 1	TEMPERATURE
	RED	2	2	100	± 2	COEFFICIENT
	ORANGE	3	3	1,000	± 3	OR METHODS
	YELLOW	4	4	10,000	± 4	OF TESTING
	GREEN	5	5	100,000	± 5	
	BLUE	6	6	1,000,000	± 6	
	VIOLET	7	7	10,000,000	± 7	
	GRAY	8	8	100,000,000	± 8	
EIA, MICA	WHITE	9	9	1,000,000,000	± 9	
	GOLD			.1		
MOLDED PAPER	SILVER			.01	± 10	
	BODY				± 20	

Figure IV-1 - 6-dot EIA and JAN color coding system for mica, and molded paper capacitors (marking system).

The 6-Dot Marking System

The 6-dot JAN and EIA systems of marking molded mica and molded paper types of capacitors are shown in Figure 1. In both the JAN and EIA 6-dot systems, the dot locations are the same, the only difference being the color of the type-dot used to indicate a molded mica capacitor. If the tolerance-dot is the same color as the body of the capacitor, the tolerance is 20%.
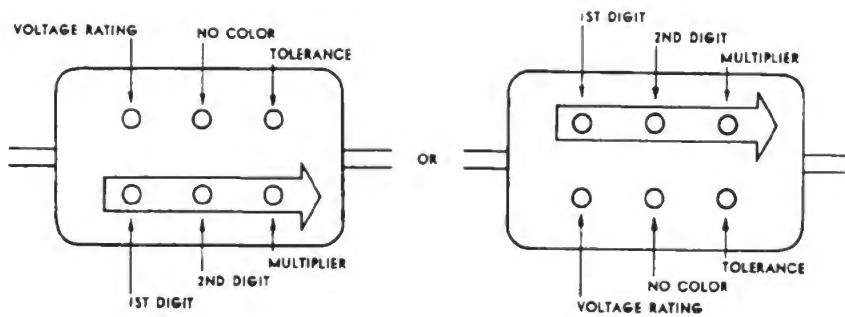
COLOR	1ST DIGIT	2ND DIGIT	MULTIPLIER	TOLERANCE (PERCENT)	VOLTAGE RATING
BLACK	0	0	1.0		
BROWN	1	1	10	± 1	100
RED	2	2	100	± 2	200
ORANGE	3	3	1,000	± 3	300
YELLOW	4	4	10,000	± 4	400
GREEN	5	5	100,000	± 5	500
BLUE	6	6	1,000,000	± 6	600
VIOLET	7	7	10,000,000	± 7	700
GRAY	8	8	100,000,000	± 8	800
WHITE	9	9	1,000,000,000	± 9	900
GOLD			.1		1000
SILVER			.01	± 10	2000
BODY				± 20	•

*WHERE NO COLOR IS INDICATED, THE VOLTAGE RATING MAY BE AS LOW AS 300 VOLTS.

Figure IV-2 - 5-dot EIA color coding system (dielectric not specified) (marking system).

The final or sixth, dot, indicating the characteristic or class of the capacitor, is a rarely used marking that pertains to the temperature coefficient or methods of testing.

The 5-Dot Marking System

The 5-dot capacitor marking system applied to molded paper and mica dielectric capacitors is shown in Figure 2. If the voltage-rating-dot is the same color as the body of the capacitor, it should be assumed that the maximum voltage that should be applied to the capacitor is 300 volts.
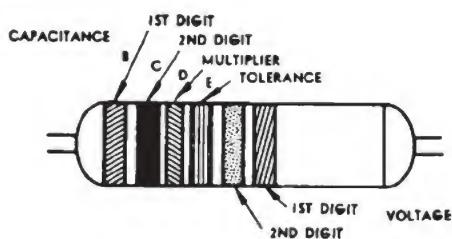
COLOR	CAPACITANCE			TOLERANCE (PERCENT)	VOLTAGE RATING	
	1ST DIGIT	2ND DIGIT	MULTIPLIER		1ST DIGIT	2ND DIGIT
BLACK	0	0	1	±20	0	0
BROWN	1	1	10		1	1
RED	2	2	100		2	2
ORANGE	3	3	1,000	±30	3	3
YELLOW	4	4	10,000	±40	4	4
GREEN	5	5	100,000	±5	5	5
BLUE	6	6	1,000,000		6	6
VIOLET	7	7			7	7
GRAY	8	8			8	8
WHITE	9	9		±10	9	9

Figure IV-3 - 6-Band EIA color coding system for tabular paper dielectric capacitors (marking system).

Tubular Paper Capacitor Marking System

The marking system and table of color values for tubular paper capacitors is shown in Figure 3. The primary difference between this system and others is that two digits or stripes are used to indicate rating. The voltage rating of the capacitor is found by taking the coded values indicated and multiplying by a constant of 100. For example, if a molded paper capacitor has color bands of red, violet, orange, black, brown, and black, its characteristics would be 27,000 picofarads, plus or minus 20 percent, with a voltage rating of 1,000 volts. If the capacitor's working voltage is less than 1,000 volts the second voltage band will be omitted.

				TOLERANCE		
COLOR	1ST DIGIT	2ND DIGIT	MULTIPLIER	MORE THAN 10 pf (IN PERCENT)	LESS THAN 10 pf (IN pf)	TEMPERATURE COEFFICIENT *
BLACK	0	0	1.0	±20	±2.0	0
BROWN	1	1	10	± 1		— 30
RED	2	2	100	± 2		— 80
ORANGE	3	3	1,000			— 150
YELLOW	4	4	10,000			— 220
GREEN	5	5		± 5	±0.5	— 330
BLUE	6	6				— 470
VIOLET	7	7				— 750
GRAY	8	8	.01		±0.25	+ 30
WHITE	9	9	.1	±10	±1.0	+ 120 TO — 750 (EIA)
						+ 500 TO — 330 (JAN)
SILVER						+ 100 (JAN)
GOLD						BYPASS OR COUPLING (EIA)

* PARTS PER MILLION PER DEGREE CENTIGRADE.

Figure IV-4 - Color coding system for ceramic capacitors having different configurations (marking system).

Ceramic Capacitor Marking System

The ceramic dielectric capacitor is usually marked with a 5-dot or fine strip code. The order of reading and location of these various dots as well as a table of values are shown in Figure 4. Since ceramic capacitors are often subjected to heat in their circuit applications, their temperature coefficient of capacitance is included in the code. It should be noted that the table has two tolerance lists — one for capacitance value less than 10 of the other for values greater than this. In the 3-dot color coding, the capacitor may have various values of tolerance, such as 2 pf, plus or minus ten percent or plus or minus 20 percent. A system has been devised to print the values of capacitance on the insulating material of disc capacitor. Quite often the letters GMV are added in print before the stamped value, these letters meaning "guaranteed minimum value".

(A)

COLOR	1ST DIGIT	2ND DIGIT	MULTIPLIER	TOLERANCE (percent)
Black	0	0	1	
Brown	1	1	10	
Red	2	2	100	
Orange	3	3	1,000	
Yellow	4	4	10,000	
Green	5	5	100,000	
Blue	6	6	1,000,000	
Violet	7	7	10,000,000	
Gray	8	8	100,000,000	
White	9	9	1,000,000,000	
Gold			.1	5
Silver			.01	10
No color				20

(B)
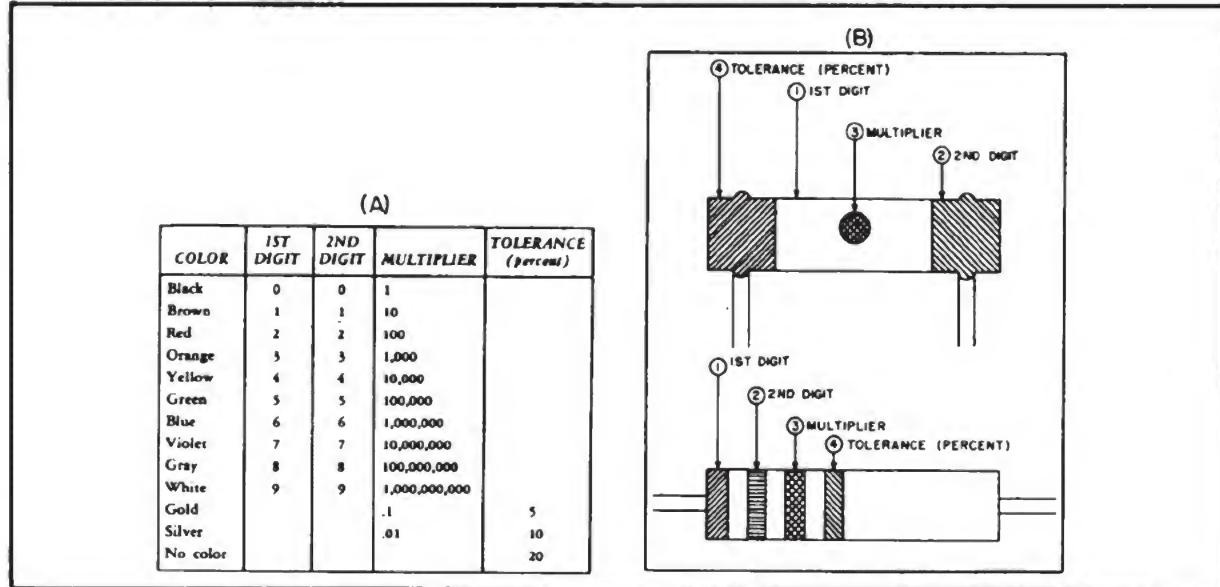
Figure 5 - EIA standard resistor color code (marking system).
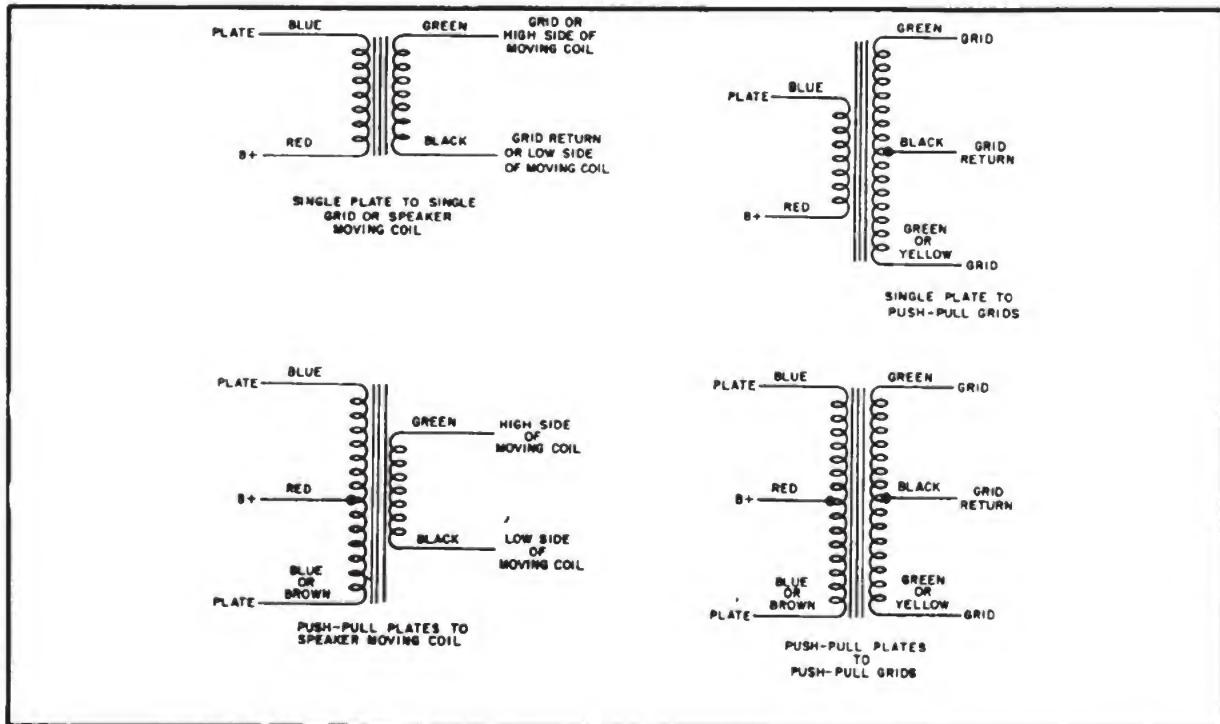
Figure 6 - Standard color coding for AF transformers.

Wirewound

These resistors are generally coated with a hard vitreous enamel and their value, in ohms, is usually printed somewhere on the resistor.

Composition

The value, in ohms, and the tolerance in percentage of a composition resistor is determined by the use of a standard color code system, (see part A of Figure 5). The first practical form of color coding was called the body-end-dot system and is shown on the component in the portion of illustration B (Figure 5).
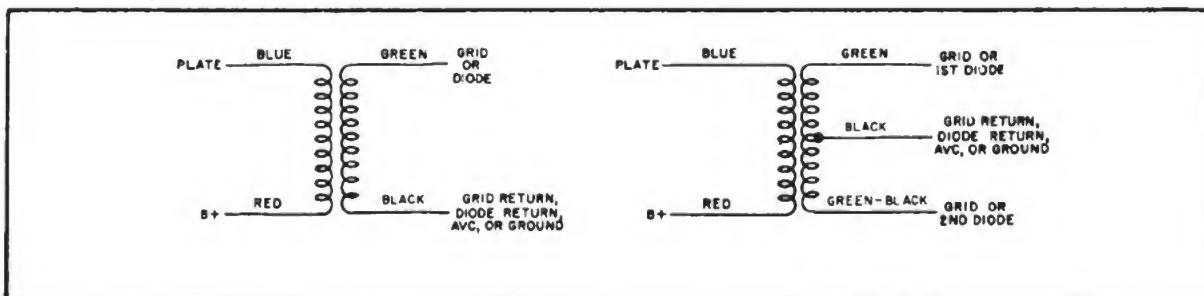
Figure IV-7 - Standard color coding for IF transformers.
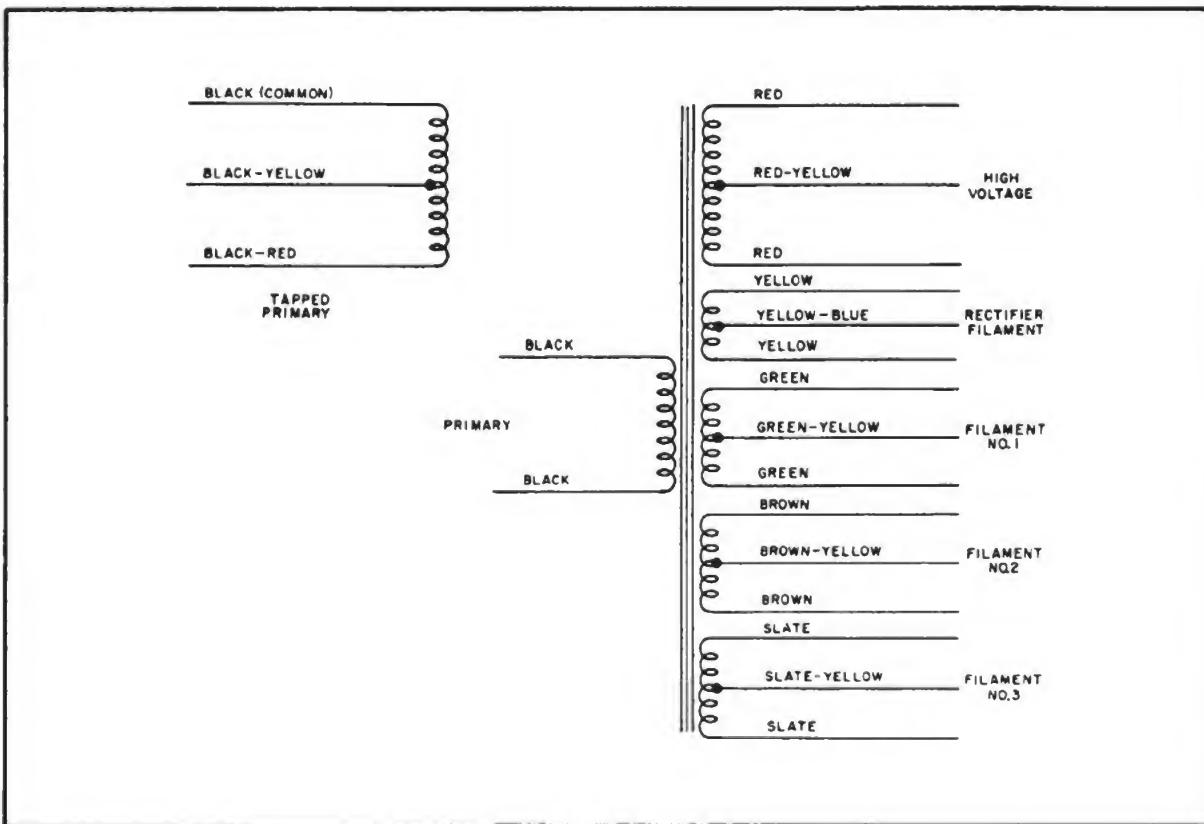
Figure IV-8 - Standard color coding for power transformers.

The present four-band system for color coding resistors, adopted by the Electronic Industries Association (EIA), is shown on the lower component in illustration B of Figure 5. The first color band indicates the value of the first digit; the second band indicates the value of the second digit; the third band indicates the multiplier, or number of zeros, to be added to the first and second digits to obtain the total resistance. When a resistor has a fourth band, it indicates the percentage of tolerance, above or below the value indicated by the other three bands. (See column 5 of Figure 5A). The absence of a fourth band (No Color) indicates the tolerance of $\pm20\%$.

Audio Frequency Transformers

Audio frequency transformers standard color coding is shown for several different designations in Figure 6.

Intermediate Frequency Transformers

Half wave and full wave IF transformers standard color coding is shown in Figure 7.

Power Transformers

The standard color coding for power transformers primary, high voltage secondary, rectifier filament and multiple sets of low voltage filament leads are shown in Figure 8. A tapped primary color coding is also shown in Figure 8.

Symbol	Type of Transmission

a. Amplitude Modulated

A0	Continuous wave, no modulation.
A1	Continuous-wave telegraphy. On-off keying.
A2	Telegraphy by keying of a modulating audio frequency. Also by keying of modulated emission.
A3	Telephony. Double sideband, full carrier.
A3a	Telephony. Single sideband, reduced carrier.
A3b	Telephony. Two independent side-bands reduced carrier.
A4	Facsimile.
A5	Television.
A9	Composite transmissions and cases not covered by above.
A9a	Composite transmissions, reduced carrier.

b. Frequency (or Phase) Modulated

F0	Absence of modulation.
F1	Telegraphy by frequency shift keying. No modulation.
F2	Telegraphy by keying of a modulating audio frequency. Also by keying of modulated emission.
F3	Telephony.
F4	Facsimile.
F5	Television.
F9	Composite transmissions and cases not covered by above.

c. Pulse Modulated

P0	Absence of modulation intended to carry information (such as radar).
P1	Telegraphy. No modulating audio frequency.
P2d	Telegraphy by keying an audio frequency modulating the pulse in amplitude.
P2e	Telegraphy by keying an audio frequency modulating the width of the pulse.
P2f	Telegraphy by keying an audio frequency modulating the phase (or position) of the pulse.
P3d	Telephony. Amplitude modulated.
P3e	Telephony. Width modulated.

P3f	Telephony. Phase (or position) modulated.
P9	Telephony. Phase (or position) modulated. Composite transmissions and cases not covered by above.

d. Classification of Time-Division Multiplex Systems

Class	Name	Code	Action of Modulating Signal
A	Pulse-time modulation	PTM	Varies some characteristic of pulse with respect to time.
	Pulse-position modulation	PPM	Varies position (phase) of pulse on time base.
	Pulse-duration modulation	PDM	Varies width of pulse (also called PWM, or Pulse-Width Modulation).
	Pulse-shape modulation		Varies shape of pulse.
	Pulse-frequency modulation	PFM	Varies pulse recurrence frequency.
B	Pulse-amplitude modulation	PAM	Varies amplitude of pulse — consists of two types: one using unipolar pulses, the other using bipolar pulses.
C	Pulse-code modulation	PCM	Varies the makeup of a series of pulses and spaces. Individual systems are classified as follows: Binary-pulse and spaces, or positive and negative pulses. Ternary-positive pulses, negative pulses, and spaces. N-ary-more complex combinations of pulses and spaces.

Appendix VIII
CONVERSION TABLES

To convert from	To	Multiply by
Abamperes	Amperes	10.0000
Abamperes	Statampere	2.998×10^{10}
Abcoulombs	Ampere-hours	2.778×10^{-3}
Abcoulombs	Coulombs	10.0000
Abcoulombs	Faradays	1.036×10^{-4}
Abcoulombs	Statcoulombs	2.998×10^{10}
Abfarads	Farads	10^{9}
Abfarads	Microfarads	10^{15}
Abfarads	Statfarads	8.988×10^{20}
Abhenrys	Henrys	10^{-9}
Abhenrys	Microhenrys	0.001
Abhenrys	Millihenrys	10^{-6}
Abhenrys	Stathenrys	1.113×10^{-21}
Abohms	Megohms	10^{-15}
Abohms	Microhms	0.001
Abohms	Ohms	10^{-9}
Abohms	Statohms	1.113×10^{-21}
Abvolts	Microvolts	0.01
Abvolts	Millivolts	10^{-5}
Abvolts	Statvolts	3.336×10^{-1}
Abvolts	Volts	10^{-8}
Acres	Ares (square dekameters)	40.46873
Acres	Hectares (square hectometers)	0.4046873
Acres	Square feet	4.356×10^{4}
Acres	Square inches	6,272,640
Acres	Square kilometers	4.047×10^{-3}
Acres	Square meters	4047
Acres	Square miles	1.563×10^{-3}
Acres	Square rods	160
Acres	Square yards	4840
Amperes	Abamperes	0.1
Amperes	Milliamperes	1000
Amperes	Statamperes	2.998×10^{9}
Ampere-hours	Abcoulombs	360
Ampere-hours	Coulombs	3600
Ampere-hours	Faradays	3.731×10^{-2}
Ampere-hours	Statcoulombs	1.080×10^{13}
Ares	Acres (US)	0.02471044
Ares	Hectares	0.01
Ares	Square feet	1076.4
Ares	Square meters	100
Ares	Square miles	3.861×10^{-5}
Ares	Square yards	119.60
Bushels (dry)	Cubic centimeters	3524×10^{4}
Bushels (dry)	Cubic feet	1.2444
Bushels (dry)	Cubic inches	2150.4

123

To convert from	To	Multiply by
Bushels (dry)	Cubic meters	3.524×10^{-2}
Bushels (dry)	Liters	35.24
Centimeters	Feet	3.281×10^{-2}
Centimeters	Inches	0.3937
Centimeters	Kilometers	10^{-5}
Centimeters	Meters	0.01
Centimeters	Mils	393.7
Centimeters	Miles	6.214×10^{-6}
Centimeters	Millimeters	10
Centimeters	Yards	1.094×10^{-2}
Centimeters/second	Feet/minute	1.969
Centimeters/second	Feet/second	3.282×10^{-2}
Centimeters/second	Kilometers/hour	0.036
Centimeters/second	Kilometers/minute	0.0006
Centimeters/second	Knots	1.943×10^{-2}
Centimeters/second	Meters/minute	0.6
Centimeters/second	Meters/second	0.01
Centimeters/second	Miles/hour	2.237×10^{-2}
Centimeters/second	Miles/minute	3.728×10^{-4}
Circular mils	Square centimeters	5.067×10^{-6}
Circular mils	Square inches	7.854×10^{-7}
Circular mils	Square millimeters	5.067×10^{-4}
Circular mils	Square mils	0.7854
Coulombs	Abcoulombs	0.1
Coulombs	Ampere-hours	2.778×10^{-4}
Coulombs	Faradays	1.036×10^{-5}
Coulombs	Statcoulombs	2.998×10^{9}
Cubic centimeters	Cubic feet	3.531×10^{-5}
Cubic centimeters	Cubic inches	6.102×10^{-2}
Cubic centimeters	Cubic meters	10^{-6}
Cubic centimeters	Cubic yards	1.308×10^{-6}
Cubic centimeters	Gallons (liquid)	2.642×10^{-4}
Cubic centimeters	Liters	0.001
Cubic centimeters	Pints (liquid)	2.113×10^{-3}
Cubic centimeters	Quarts (liquid)	1.057×10^{-3}
Cubic feet	Bushels (dry)	0.8036
Cubic feet	Cubic centimeters	2.832×10^{4}
Cubic feet	Cubic inches	1728
Cubic feet	Cubic meters	2.832×10^{-2}
Cubic feet (US)	Cubic yards	3.704×10^{-2}
Cubic feet	Gallons (liquid)	7.481
Cubic feet	Liters	28.316
Cubic feet	Pints (liquid)	59.84
Cubic feet	Quarts (liquid)	29.922
Cubic hectometers	Cubic meters	10^{6}
Cubic inches	Bushels (dry)	4.6503×10^{-4}
Cubic inches	Cubic centimeters	16.39
Cubic inches	Cubic feet	5.787×10^{-4}
Cubic inches	Cubic meters	1.639×10^{-5}
Cubic inches (US)	Cubic yards	2.143×10^{-5}
Cubic inches	Gallons	4.329×10^{-3}
Cubic inches	Liters	1.639×10^{-2}
Cubic inches	Pints (liquid)	3.463×10^{-2}
Cubic inches	Quarts (liquid)	1.732×10^{-2}
Cubic meters	Bushels (dry)	28.38
Cubic meters	Cubic centimeters	10^{6}

To convert from	To	Multiply by
Cubic meters	Cubic feet	35.31
Cubic meters	Cubic inches	6.102×10^4
Cubic meters	Cubic yards	1.308
Cubic meters	Gallons (liquid)	264.2
Cubic meters	Liters	1000
Cubic meters	Pints (liquid)	2113
Cubic meters	Quarts (liquid)	1057
Cubic meters	Steres	1
Cubic yards	Cubic centimeters	7.646×10^5
Cubic yards	Cubic feet	27
Cubic yards	Cubic inches	46656
Cubic yards	Cubic meters	0.7646
Cubic yards	Gallons	202.0
Cubic yards	Liters	764.6
Cubic yards	Pints (liquid)	1616
Cubic yards	Quarts (liquid)	807.9
Decimeters	Meters	0.1
Decigrams	Grams	0.1
Decisteres	Cubic meters	0.1
Degrees	Circumferences (revolutions)	2.778×10^{-3}
Degrees	Minutes	60
Degrees	Quadrants	1.111×10^{-2}
Degrees	Radians	1.745×10^{-2}
Degrees	Seconds	3600
Degrees/second	Radians/second	1.745×10^{-2}
Degrees/second	Revolutions/minute	0.1667
Degrees/second	Revolutions/second	2.778×10^{-3}
Dekagrams	Grams	10
Dekameters	Meters	10
Faradays	Abcoulombs	9649
Faradays	Ampere-hours	26.81
Faradays	Coulombs	9.649×10^4
Faradays	Statcoulombs	2.893×10^{14}
Farads	Abfarads	10^{-9}
Farads	Microfarads	10^6
Farads	Statfarads	8.988×10^{11}
Feet	Centimeters	30.48
Feet	Inches	12
Feet	Kilometers	3.048×10^{-4}
Feet	Meters	0.3048
Feet	Miles (nautical)	1.645×10^{-4}
Feet	Miles (statute)	1.894×10^{-4}
Feet	Mils	1.2×10^4
Feet	Millimeters	304.8
Feet	Yards	0.3333
Feet/minute	Centimeter/second	0.5080
Feet/minute	Feet/second	1.667×10^{-2}
Feet/minute	Kilometers/hour	1.829×10^{-2}
Feet/minute	Kilometers/second	3.048×10^{-4}
Feet/minute	Knots	9.868×10^{-3}
Feet/minute	Meters/minute	0.3048
Feet/minute	Meters/second	5.080×10^{-3}
Feet/minute	Miles/hour	1.136×10^{-2}
Feet/minute	Miles/minute	1.894×10^{-4}
Feet/second	Centimeters/second	30.48
Feet/second	Feet/minute	60

To convert from	To	Multiply by
Feet/second	Kilometers/hour	1.097
Feet/second	Kilometers/minute	1.829×10^{-2}
Feet/second	Knots	0.5921
Feet/second	Meters/minute	18.29
Feet/second	Meters/second	0.3048
Feet/second	Miles/hour	0.6818
Feet/second	Miles/minute	1.136×10^{-2}
Gallons (liquid)	Cubic centimeters	3785.
Gallons (liquid)	Cubic feet	0.1337
Gallons (liquid)	Cubic inches	231
Gallons (liquid)	Cubic meters	3.785×10^{-3}
Gallons (liquid)	Cubic yards	4.951×10^{-3}
Gallons (liquid)	Liters	3.785
Gallons (liquid)	Pints (liquid)	8
Gallons (liquid)	Quarts (liquid)	4
Grains	Grams	6.480×10^{-2}
Grains	Kilograms	6.481×10^{-5}
Grains	Milligrams	64.81
Grains	Ounces (avoirdupois)	2.286×10^{-3}
Grains	Pounds (avoirdupois)	1.429×10^{-4}
Grams	Grains	15.43
Grams	Kilograms	6.480×10^{-5}
Grams	Milligrams	64.80
Grams	Ounces (avoirdupois)	3.527×10^{-2}
Grams	Pounds (avoirdupois)	2.205×10^{-3}
Grams	Tons (long)	9.842×10^{-7}
Grams	Tons (metric)	10^{-6}
Grams	Tons (short)	1.102×10^{-6}
Hectares	Acres	2.471
Hectares	Acres	100
Hectares	Square feet	1.076×10^{5}
Hectares	Square meters	10000
Hectares	Square rods	3.954×10^{2}
Hectares	Square yards	11959.85
Hectograms	Grams	100
Hectograms	Ounces (avoirdupois)	3.527
Hectoliters	Liters	100
Hectometers	Meters	100
Hectometers	Rods	19.88
Hectometers	Yards	109.4
Hectowatts	Watts	100
Hemispheres	Spheres	0.5
Hemispheres	Spherical right angles	4
Hemispheres	Steradians	6.283
Henrys	Abhenrys	10^{9}
Henrys	Microhenrys	10^{6}
Henrys	Millihenrys	1000
Henrys	Stathenrys	1.113×10^{-12}
Inches	Centimeters	2.540
Inches	Feet	8.333×10^{-2}
Inches	Kilometers	2.540×10^{-5}
Inches	Meters	2.540×10^{-2}
Inches	Miles	1.578×10^{-5}
Inches	Millimeters	25.40
Inches	Mils	1000
Inches	Yards	2.778×10^{-2}

To convert from	To	Multiply by
Kilograms	Grains	1.543×10^4
Kilograms	Grams	1000
Kilograms	Milligrams	10^6
Kilograms	Ounces (avoirdupois)	35.27
Kilograms	Pounds (avoirdupois)	2.205
Kilograms	Tons (long)	9.842×10^{-4}
Kilograms	Tons (metric)	0.001
Kilograms	Tons (short)	1.102×10^{-3}
Kiloliters	Gallons (liquid)	264.18
Kiloliters	Liters	1000
Kilometers	Centimeters	10^5
Kilometers	Feet	3281
Kilometers	Inches	3.937×10^4
Kilometers	Meters	1000
Kilometers	Miles (nautical)	0.5396
Kilometers	Miles (statute)	0.6214
Kilometers	Millimeters	10^6
Kilometers	Mils	3.937×10^7
Kilometers	Yards	1094
Kilometers/hour	Centimeters/second	27.78
Kilometers/hour	Feet/minute	54.68
Kilometers/hour	Feet/second	0.9113
Kilometers/hour	Kilometers/minute	1.667×10^{-2}
Kilometers/hour	Knots	0.5396
Kilometers/hour	Meters/minute	16.67
Kilometers/hour	Meters/second	0.2778
Kilometers/hour	Miles/hour	0.6214
Kilometers/hour	Miles/minute	1.036×10^{-2}
Kilometers/minute	Centimeters/second	1667
Kilometers/minute	Feet/minute	3281
Kilometers/minute	Feet/second	54.68
Kilometers/minute	Kilometers/hour	60
Kilometers/minute	Knots	32.38
Kilometers/minute	Meters/minute	1000
Kilometers/minute	Meters/second	16.67
Kilometers/minute	Miles/hour	37.28
Kilometers/minute	Miles/minute	0.6214
Kilowatt hours	Watt-hours	1000
Kilowatts	Watts	1000
Knots	Centimeters/second	51.48
Knots	Feet/hour	6080.20
Knots	Feet/minute	101.3
Knots	Feet/second	1.689
Knots	Kilometers/hour	1.853
Knots	Kilometers/minute	3.088×10^{-2}
Knots	Meters/minute	30.88
Knots	Meters/second	0.5148
Knots	Miles/hour	1.152
Knots	Miles/minute	1.919×10^{-2}
Liters	Bushels (dry)	2.838×10^{-2}
Liters	Cubic centimeters	1000
Liters	Cubic feet	3.531×10^{-2}
Liters	Cubic inches	61.02
Liters	Cubic meters	0.001
Liters	Cubic yards	1.308×10^{-3}
Liters	Gallons (liquid)	0.2642

To convert from	To	Multiply by
Liters	Pints (liquid)	2.113
Liters	Quarts (liquid)	1.057
Megacycles	Cycles	10^6
Megameters	Meters	10^6
Megohms	Abohms	0.001
Megohms	Abohms	10^{15}
Megohms	Microhms	10^{12}
Megohms	Ohms	10^6
Megohms	Statohms	1.112×10^{-6}
Meters	Centimeters	100
Meters	Feet	3.281
Meters	Inches	39.37
Meters	Kilometers	0.001
Meters	Megameters	10^{-6}
Meters	Miles (statute)	6.214×10^{-4}
Meters	Millimeters	1000
Meters	Millimicrons	10^9
Meters	Mils	3.937×10^4
Meters	Yards	1.094
Meters/minute	Centimeters/second	1.667
Meters/minute	Feet/minute	3.281
Meters/minute	Feet/second	5.468×10^{-2}
Meters/minute	Kilometers/hour	0.06
Meters/minute	Kilometers/minute	0.001
Meters/minute	Knots	3.238×10^{-2}
Meters/minute	Meters/second	1.667×10^{-2}
Meters/minute	Miles/hour	3.728×10^{-2}
Meters/minute	Miles/minute	6.214×10^{-4}
Meters/second	Centimeters/second	100
Meters/second	Feet/minute	196.8
Meters/second	Feet/second	3.281
Meters/second	Kilometers/hour	3.6
Meters/second	Kilometers/minute	0.06
Meters/second	Knots	1.943
Meters/second	Meters/minute	60
Meters/second	Miles/hour	2.237
Meters/second	Miles/minute	3.728×10^{-2}
Microfarads	Abfarads	10^{-15}
Microfarads	Farads	10^{-6}
Microfarads	Statfarads	8.988×10^5
Micrograms	Grams	10^{-6}
Milliograms	Milligrams	0.001
Microhenrys	Abhenrys	1,000
Microhenrys	Henrys	10^{-6}
Microhenrys	Millihenrys	0.001
Microhenrys	Stathenrys	1.113×10^{-18}
Microhms	Abohms	1000
Microhms	Megohms	10^{-12}
Microhms	Ohms	10^{-6}
Microhms	Statohms	1.113×10^{-18}
Microliters	Liters	10^{-6}
Micromicrofarads	Farads	10^{-12}
Microvolts	Abvolts	100
Microvolts	Millivolts	0.001
Microvolts	Statvolts	3.336×10^{-9}
Microvolts	Volts	10^{-6}

To convert from	To	Multiply by
Miles	Centimeters	1.609×10^5
Miles	Feet	5280
Miles	Inches	6.336×10^4
Miles	Kilometers	1.609
Miles	Meters	1609
Miles	Miles (nautical)	0.8684
Miles	Rods	320
Miles	Yards	1760
Miles/hour	Centimeters/second	44.70
Miles/hour	Feet/minute	88
Miles/hour	Feet/second	1.467
Miles/hour	Kilometers/hour	1.609
Miles/hour	Kilometers/minute	2.682×10^{-2}
Miles/hour	Knots	0.8684
Miles/hour	Meters/minute	26.82
Miles/hour	Meters/second	0.4470
Miles/hour	Miles/minute	1.667×10^{-2}
Miles/minute	Centimeters/second	2682
Miles/minute	Feet/minute	5280
Miles/minute	Feet/second	88
Miles/minute	Kilometers/hour	96.54
Miles/minute	Kilometers/minute	1.609
Miles/minute	Knots	52.10
Miles/minute	Meters/minute	1609
Miles/minute	Meters/second	26.82
Miles/minute	Miles/hour	60
Milligrams	Grains	1.543×10^{-2}
Milligrams	Grams	0.001
Milligrams	Kilograms	10^{-6}
Milligrams	Ounces (avoirdupois)	3.527×10^{-5}
Milligrams	Pounds (avoirdupois)	2.205×10^{-6}
Milligrams	Tons (long)	9.842×10^{-10}
Milligrams	Tons (metric)	10^{-9}
Milligrams	Tons (short)	1.102×10^{-9}
Millihenrys	Abhenrys	10^6
Millihenrys	Henrys	0.001
Millihenrys	Microhenrys	1000
Millihenrys	Stathenrys	1.112×10^{-15}
Millileters	Liters	0.001
Millimeters	Centimeters	0.1
Millimeters	Feet	3.281×10^{-3}
Millimeters	Inches	3.937×10^{-2}
Millimeters	Kilometers	10^{-6}
Millimeters	Meters	0.001
Millimeters	Miles	6.214×10^{-7}
Millimeters	Mils	39.37
Millimeters	Yards	1.094×10^{-3}
Millimicrons	Microns	0.001
Millivolts	Abvolts	10^5
Millivolts	Microvolts	1000
Millivolts	Statvolts	3.336×10^{-6}
Millivolts	Volts	0.001
Mils	Centimeters	2.540×10^{-3}
Mils	Feet	8.333×10^{-5}
Mils	Inches	0.001
Mils	Kilometers	2.540×10^{-8}

To convert from	To	Multiply by
Mils	Millimeters	2.540×10^{-2}
Mils	Yards	2.778×10^{-5}
Minutes (angle)	Degrees	1.667×10^{-2}
Minutes (angle)	Quadrants	1.852×10^{-4}
Minutes (angle)	Radians	2.909×10^{-4}
Minutes (angle)	Revolutions (circumferences)	4.630×10^{-5}
Minutes (angle)	Seconds	60
Myriagrams	Grams	10,000
Myriagrams	Kilograms	10
Myriameters	Kilometers	10
Myriameters	Meters	10,000
Myriameters	Miles	6,21370
Ohms	Abohms	10^9
Ohms	Megohms	10^{-6}
Ohms	Microhms	10^6
Ohms	Statohms	1.112×10^{-12}
Ounces (avoirdupois)	Grains	437.5
Ounces (avoirdupois)	Grams	28.35
Ounces (avoirdupois)	Kilograms	$2.835. \times 10^{-2}$
Ounces (avoirdupois)	Milligrams	2.835×10^4
Ounces (avoirdupois)	Pounds (avoirdupois)	6.250×10^{-2}
Ounces (avoirdupois)	Tons (long)	2.790×10^{-5}
Ounces (avoirdupois)	Tons (metric)	2.835×10^{-5}
Ounces (avoirdupois)	Tons (short)	3.125×10^{-5}
Pints (liquid)	Cubic centimeters	473.2
Pints (liquid)	Cubic feet	1.671×10^{-2}
Pints (liquid)	Cubic inches	28.87
Pints (liquid)	Cubic meters	4.732×10^{-4}
Pints (liquid)	Cubic yards	6.189×10^{-4}
Pints (liquid)	Gallons (liquid)	0.125
Pounds (avoirdupois)	Grains	7000
Pounds (avoirdupois)	Grams	453.6
Pounds (avoirdupois)	Kilograms	0.4536
Pounds (avoirdupois)	Milligrams	4.536×10^5
Pounds (avoirdupois)	Ounces (avoirdupois)	16
Pounds (avoirdupois)	Tons (long)	4.464×10^{-4}
Pounds (avoirdupois)	Tons (short)	0.0005
Quadrants	Degrees	90
Quadrants	Minutes	5400
Quadrants	Radians	1.571
Quadrants	Revolutions (circumferences)	0.25
Quadrants	Seconds	3.24×10^5
Quarts (liquid)	Cubic centimeters	946.4
Quarts (liquid)	Cubic feet	3.342×10^{-2}
Quarts (liquid)	Cubic inches	57.75
Quarts (liquid)	Cubic meters	9.464×10^{-4}
Quarts (liquid)	Cubic yards	1.238×10^{-3}
Quarts (liquid)	Gallons (liquid)	0.25
Radians	Circumferences	0.1591
Radians	Degrees	57.30
Radians	Degrees, minutes, seconds	$57°, 17', 44.8''$
Radians	Minutes	3438
Radians	Quadrants	0.6366
Radians	Revolutions	0.1591
Radians	Seconds	2.063×10^5
Radians/second	Degrees/second	57.30

To convert from	To	Multiply by
Radians/second	Revolutions/minute	9.549
Radians/second	Revolutions/second	0.1592
Revolutions (circumferences)	Degrees	360
Revolutions (circumferences)	Minutes	2.16×10^4
Revolutions (circumferences)	Quadrants	4
Revolutions (circumferences)	Radians	6.283
Revolutions (circumferences)	Seconds	1.296×10^6
Revolutions/minute	Degrees/second	6
Revolutions/minute	Radians/second	0.1047
Revolutions/minute	Revolutions/second	1.667×10^{-2}
Revolutions/second	Degrees/second	360
Revolutions/second	Radians/second	6.283
Revolutions/second	Revolutions/minute	60
Seconds (angle)	Degrees	2.778×10^{-4}
Seconds (angle)	Minutes	1.667×10^{-2}
Seconds (angle)	Quadrants	3.087×10^{-6}
Seconds (angle)	Radians	4.848×10^{-6}
Seconds (angle)	Revolutions (circumferences)	7.716×10^{-7}
Spheres	Hemispheres	2
Spheres	Spherical right angles	8
Spheres	Steradians	12.57
Spherical right angles	Hemispheres	0.25
Spherical right angles	Spheres	0.125
Spherical right angles	Steradians	1.571
Square centimeters	Circuilar mils	1.973×10^5
Square centimeters	Square decimeters	0.01
Square centimeters	Square feet	1.076×10^{-3}
Square centimeters	Square inches	0.1550
Square centimeters	Square kilometers	10^{-10}
Square centimeters	Square meters	0.0001
Square centimeters	Square miles	3.861×10^{-11}
Square centimeters	Square millimeters	100
Square centimeters	Square yards	1.196×10^{-4}
Square feet	Acres	2.296×10^{-5}
Square feet	Acres	9.290×10^{-4}
Square feet	Circular mils	1.833×10^8
Square feet	Square centimeters	929.0
Square feet	Square inches	144
Square feet	Square kilometers	9.290×10^{-8}
Square feet	Square meters	9.290×10^{-2}
Square feet	Square miles	3.587×10^{-8}
Square feet	Square millimeters	9.290×10^4
Square inches	Circular mils	1.273×10^6
Square inches	Square centimeters	6.452
Square inches	Square feet	6.944×10^{-3}
Square inches	Square kilometers	6.452×10^{-10}
Square inches	Square meters	6.452×10^{-4}
Square inches	Square millimeters	645.2
Square inches	Square yards	7.716×10^{-4}
Square kilometers	Acres	247.1
Square kilometers	Square centimeters	10^{10}
Square kilometers	Square feet	1.076×10^7
Square kilometers	Square inches	1.550×10^9
Square kilometers	Square meters	10^6
Square kilometers	Square miles	0.3861
Square kilometers	Square millimeters	10^{12}

To convert from	To	Multiply by
Square kilometers	Square yards	1.196×10^6
Square meters	Acres	2.471×10^{-4}
Square meters	Acres	0.01
Square meters	Circular mils	1.973×10^9
Square meters	Square centimeters	10^4
Square meters	Square feet	10.76
Square meters	Square inches	1550
Square meters	Square kilometers	10^{-6}
Square meters	Square miles	3.861×10^{-7}
Square meters	Square millimeters	10^6
Square meters	Square yards	1.196
Square miles	Acres	640
Square miles	Square centimeters	2.590×10^{10}
Square miles	Square feet	2.788×10^7
Square miles	Square inches	4.015×10^9
Square miles	Square kilometers	2.590
Square miles	Square meters	2.590×10^6
Square miles	Square yards	3.098×10^6
Square millimeters	Circular mils	1973
Square millimeters	Square centimeters	0.01
Square millimeters	Square feet	1.076×10^{-5}
Square millimeters	Square inches	1.550×10^{-3}
Square millimeters	Square kilometers	10^{-12}
Square millimeters	Square meters	10^{-6}
Square millimeters	Square miles	3.861×10^{-13}
Square millimeters	Square yards	1.196×10^{-6}
Square rods	Acres	0.00625
Square rods	Square feet	272.25
Square rods	Square inches	39204
Square rods	Square meters	25.293
Square rods	Square miles	9.766×10^{-6}
Square rods	Square yards	30.25
Square yards	Acres	2.066×10^{-4}
Square yards	Square centimeters	8361
Square yards	Square feet	9
Square yards	Square inches	1296
Square yards	Square kilometers	8.361×10^{-7}
Square yards	Square meters	0.8361
Square yards	Square miles	3.228×10^{-7}
Square yards	Square millimeters	8.361×10^{-5}
Statamperes	Abamperes	3.335×10^{-11}
Statamperes	Amperes	3.335×10^{-10}
Statcoulombs	Abcoulombs	3.335×10^{-11}
Statcoulombs	Ampere-hours	9.259×10^{-14}
Statcoulombs	Coulombs	3.335×10^{-10}
Statcoulombs	Faradays	3.457×10^{-15}
Statfarads (or centimeters)	Abfarads	1.112×10^{-21}
Statfarads	Farads	1.112×10^{-12}
Statfarads	Microfarads	1.112×10^{-6}
Stathenrys	Abhenrys	8.988×10^{20}
Stathenrys	Henrys	8.988×10^{11}
Stathenrys	Microhenrys	8.988×10^{17}
Stathenrys	Millihenrys	8.988×10^{14}
Statohms	Abohms	8.988×10^{20}
Statohms	Megohms	8.988×10^5
Statohms	Microhms	8.988×10^{17}

To convert from	To	Multiply by
Statohms	Ohms	8.988×10^{11}
Statvolts	Abvolts	2.998×10^{10}
Statvolts	Microvolts	2.998×10^{8}
Statvolts	Millivolts	2.998×10^{5}
Statvolts	Volts	299.8
Steradians	Hemispheres	0.1592
Steradians	Spheres	7.958×10^{-2}
Steradians	Spherical right angles	0.6366
Steres	Cubic meters	1
Steres	Liters	999.973
Tons (long)	Grams	1.016×10^{6}
Tons (long)	Kilograms	1016
Tons (long)	Milligrams	1.016×10^{9}
Tons (long)	Ounces (avoirdupois)	3.584×10^{4}
Tons (long)	Pounds (avoirdupois)	2240
Tons (long)	Tons (metric)	1.016
Tons (long)	Tons (short)	1.120
Tons (metric)	Grams	10^{6}
Tons (metric)	Kilograms	1000
Tons (metric)	Milligrams	10^{9}
Tons (metric)	Ounces (avoirdupois)	3.527×10^{4}
Tons (metric)	Pounds (avoirdupois)	2205
Tons (metric)	Tons (long)	0.9842
Tons (metric)	Tons (short)	1.102
Tons (short)	Grams	9.072×10^{5}
Tons (short)	Kilograms	907.2
Tons (short)	Milligrams	9.072×10^{8}
Tons (short)	Ounces (avoirdupois)	3.2×10^{4}
Tons (short)	Pounds (avoirdupois)	2000
Tons (short)	Tons (long)	0.8929
Tons (short)	Tons (metric)	0.9072
Volts	Abvolts	10^{8}
Volts	Microvolts	10^{6}
Volts	Millivolts	1000
Volts	Statvolts	3.335×10^{-3}
Watts	Horsepower	0.0013410
Watts	Kilowatts	0.001
Yards	Centimeters	91.44
Yards	Feet	3
Yards	Inches	36
Yards	Kilometers	9.144×10^{-4}
Yards	Meters	0.9144
Yards	Miles	5.682×10^{-4}
Yards	Miles (nautical)	4.934×10^{-4}
Yards	Millimeters	914.4
Yards	Mils	3.6×10^{4}

A_{hf} . . . . High frequency gain
A_{lf} . . . . Low frequency gain
A_v . . . . Voltage gain
C_c . . . . Coupling capacitor
C_d . . . . Distributed capacitance
C_{gk} . . . . Grid-to-cathode capacitance
C_{gp} . . . . Grid-to-plate capacitance
C_i . . . . Input capacitance
C_k . . . . Cathode bypass capacitor
C_o . . . . Output capacitance
C_{pk} . . . . Plate-to-cathode capacitance
C_s . . . . Shunt capacitance $(C_d + C_i + C_o)$
E_b . . . . Plate volts (dc)
E_{bb} . . . . Supply volts (dc)
E_{bo} . . . . Quiescent plate voltage
E_{c1} . . . . Control grid voltage
E_{c2} . . . . Screen grid voltage
E_{cc} . . . . Control grid supply voltage
E_f . . . . Filament terminal voltage
e_b . . . . Instantaneous total plate volts (ac and dc)
e_{c1} . . . . Instantaneous total control grid volts (ac and dc)
e_{c2} . . . . Instantaneous total screen grid volts (ac and dc)
e_{g1} . . . . Instantaneous value of ac control grid volts
e_{g2} . . . . Instantaneous value of ac screen grid volts
e_{po} . . . . Instantaneous value of plate voltage above and below the quiescent value
E_g . . . . RMS value of grid volts
E_p . . . . RMS value of plate volts
gm . . . . Grid-plate transconductance (mutual conductance)

I_b . . . . DC value of plate volts
I_{bo} . . . . . . Quiescent value of plate current
I_{c1} . . . . . . DC value of control grid current
I_{c2} . . . . . . DC value of screen grid current
I_f . . . . Filament or heater current
I_{g1} . . . . . . RMS value of control grid current
I_{g2} . . . . . . RMS value of screen grid current
I_{gm1} . . . . . Crest values of ac current control grid
I_{gm2} . . . . . Crest values of ac current screen grid
I_p . . . . RMS values of plate current
I_{pm} . . . . . . Crest value of plate current
I_s . . . . Total electron emission
i_b . . . . Instantaneous total value of plate current
i_{c1} . . . . . . Instantaneous total value of control grid current
i_{c2} . . . . . . Instantaneous total value of screen grid current
i_{g1} . . . . . . Instantaneous ac value of control grid current
i_{g2} . . . . . . Instantaneous ac value of screen grid current
i_p . . . . Instantaneous ac value of plate current
i_{po} . . . . . . Instantaneous values of plate current above and below the quiescent value
R_b . . . . DC plate resistance
R_g . . . . DC grid resistance
R_k . . . . DC cathode resistance
RL . . . . Plate load resistance
r_p . . . . AC plate resistance
μ . . . . Amplification factor

Appendix X

TRANSISTOR SYMBOLS

Semiconductor, General
BV . . . . . Breakdown voltage
T_A . . . . Ambient temperature
T_{ep} . . . . Operating temperature

Transistor
B, b . . . . Base electrode
C, c . . . . Collector electrode
C_{ib} . . . . Input capacitance (common base)
C_{ie} . . . . Input capacitance (common emitter)
C_{ob} . . . . Output capacitance (common base)
C_{oe} . . . . Output capacitance (common emitter)
E, e . . . . Emitter electrode
I_B . . . . . Base current (dc)
i_b . . . . . . Base current (instantaneous)
I_C . . . . . Collector current (dc)
i_c . . . . . . Collector current (instantaneous)
I_{CBO} . . . Collector cutoff current (dc)
    emitter open

I_{CEO} . . . Collector cutoff current (dc)
    base open
I_E . . . . . Emitter current
R_B . . . . . External base resistance
r_b . . . . . Base spreading resistance
r_i . . . . . . Input junction resistance
V_{BB} . . . . Base supply voltage
V_C . . . . . Collector voltage (with respect to
    ground or common point)
V_{BE} . . . Base to emitter voltage (dc)
V_{CB} . . . Collector to base voltage (dc)
V_{CE} . . . Collector to emitter voltage (dc)
V_{ce} . . . . Collector to emitter voltage (rms)
v_{ce} . . . . Collector to emitter voltage
    (instantaneous)
V_{CE} . . . . Collector to emitter saturation
 (sat)   voltage
V_{EBO} . . Emitter to base voltage (static)
V_{CC} . . . Collector supply voltage
V_{EE} . . . Emitter supply voltage

ACORN TUBE. An acorn-shaped vacuum tube designed for ultra-high-frequency circuits. The tube has short electron transit time and low interelectrode capacitance because of close spacing and small size electrodes.

ALIGN. To adjust the tuned circuits of a receiver or transmitter for maximum signal response.

ALTERNATION. One-half of a complete cycle.

AMMETER. An instrument for measuring the electron flow in amperes.

AMPERE (amp). The basic unit of current or electron flow.

AMPLIFICATION (A). The process of increasing the strength of a signal.

AMPLIFICATION FACTOR (μ). The ratio of a small change in plate voltage to a small change in grid voltage, with all other electrode voltages constant, required to produce the same small change in plate current.

AMPLIFIER. A device used to increase the signal voltage, current, or power, generally composed of a vacuum tube or semiconductor device and associated circuit called a stage. It may contain several stages in order to obtain a desired gain.

AMPLITUDE. The maximum instantaneous value of an alternating voltage or current, measured in either the positive or negative direction.

AMPLITUDE DISTORTION. The changing of a waveshape so that it is no longer proportional to its original form. Also known as harmonic distortion.

ANODE. A positive electrode; the plate of a vacuum tube.

ANTENNA. A device used to radiate or absorb RF energy.

AQUADAG. A graphite coating on the inside of certain cathode-ray tubes for collecting secondary electrons emitted by the screen.

ARRAY (antenna). An arrangement of antenna elements, usually dipoles, which results in desirable directional characteristics.

ATTENUATION. The reduction in the strength of a signal.

AUDIO FREQUENCY (AF). A frequency which can be detected as a sound by the human ear. The range of audio frequencies extends approximately from 20 to 20,000 cycles per second.

AUTODYNE CIRCUIT. A circuit in which the same elements and vacuum tube are used as an oscillator and as a detector. The output has a frequency equal to the difference between the frequencies of the received signal and the oscillator signal.

AUTOMATIC GAIN CONTROL (agc). A method of automatically regulating the gain of a receiver so that the output tends to remain constant though the incoming signal may vary in strength.

AUTOMATIC VOLUME CONTROL (avc). See Automatic Gain Control.

AUTOTRANSFORMER. A transformer in which part of the primary winding is used as a secondary winding, or vice versa.

AZIMUTH. The angular measurement in a horizontal plane and in a clockwise direction, beginning at a point oriented to north.

BAND OF FREQUENCIES. The frequencies existing between two definite limits.

BAND-PASS FILTER. A circuit designed to pass with nearly equal response all currents having frequencies within a definite band, and to reduce substantially the amplitudes of currents of all frequencies outside that band.

BAZOOKA. See Line-Balance Converter.

BEAM-POWER TUBE. A high vacuum tube in which the electron stream is directed in concentrated beams from the cathode to the plate. Variously termed beam-power tetrode and beam-power pentode.

BEAT FREQUENCY. A frequency resulting from the heterodyning of two different frequencies. It is numerically equal to the difference between or the sum of these two frequencies.

BEAT NOTE. See Beat Frequency.

BIAS. The average dc difference of potential between the cathode and control grid of a vacuum tube.

BIASING RESISTOR. A resistor used to provide the voltage drop for a required bias.

BLANKING. See Gating.

BLEEDER. A resistance connected in parallel with a power supply output to protect equipment from excessive voltages if the load is removed or substantially reduced; to improve the voltage regulation, and to drain the charge

remaining in the filter capacitors when the unit is turned off.

BLOCKING CAPACITOR. A capacitor used to block the flow of direct current while permitting the flow of alternating current.

BREAK-DOWN VOLTAGE. The voltage at which an insulator or dielectric ruptures, or at which ionization and conduction take place in a gas or vapor.

BRILLIANCE MODULATION. See Intensity Modulation.

BUFFER AMPLIFIER. An amplifier used to isolate the output of an oscillator from the effects produced by changes in voltage or loading in following circuits.

BUNCHER. The electrode of a velocity-modulated tube which alters the velocity of electrons in the constant current beam causing the electrons to become bunched in a drift space beyond the buncher electrode.

BYPASS CAPACITOR. A capacitor used to provide an alternating current path of comparatively low impedance around a circuit element.

CAPACITANCE. The property of two or more bodies which enables them to store electrical energy in an electrostatic field between the bodies.

CAPACITIVE COUPLING. A method of transferring energy from one circuit to another by means of a capacitor that is common to both circuits.

CAPACITIVE REACTANCE (X_c). The opposition offered to the flow of an alternating current by capacitance, expressed in ohms.

CAPACITOR. Two electrodes or sets of electrodes in the form of plates, separated from each other by an insulating material called the dielectric.

CARRIER. The RF component of a transmitted wave upon which an audio signal or other form of intelligence can be impressed.

CATCHER. The electrode of a velocity-modulated tube which receives energy from the bunched electrons.

CATHODE (K). The electrode in a vacuum tube which is the source of electron emission. Also a negative electrode.

CATHODE BIAS. The method of biasing a tube by placing the biasing resistor in the common cathode return circuit, making the cathode more positive, rather than the grid more negative, with respect to ground.

CATHODE FOLLOWER. A vacuum tube circuit in which the input signal is applied between the control grid and ground, and the output is taken from the cathode and ground. A cathode follower has a high input impedance and a low output impedance.

CHARACTERISTIC IMPEDANCE (Z_0). The ra-

tio of the voltage to the current at every point along a transmission line on which there are no standing waves.

CHOKE. A coil which impedes the flow of alternating current of a specified frequency range because of its high inductive reactance at that range.

CHOPPING. See Limiting.

CLAMPING CIRCUIT. A circuit which maintains either amplitude extreme of a waveform at a certain level of potential.

CLASS A OPERATION. Operation of a vacuum tube so that plate current flows throughout the entire operating cycle and distortion is kept to a minimum.

CLASS AB OPERATION. Operation of a vacuum tube with grid bias so that the operating point is approximately halfway between Class A and Class B.

CLASS B OPERATION. Operation of a vacuum tube with bias at or near cut-off so that plate current flows during approximately one-half cycle.

CLASS C OPERATION. Operation of a vacuum tube with bias considerably beyond cut-off so that plate current flows for less than one-half cycle.

CLIPPING. See Limiting.

COAXIAL CABLE. A transmission line consisting of two conductors concentric with, and insulated from each other.

COEFFICIENT OF COUPLING (K). A numerical indication of the degree of coupling existing between two circuits, expressed in terms of either a decimal or a percentage.

CONDENSER. See Capacitor.

CONDUCTANCE (G). The ability of a material to conduct or carry an electric current. It is the reciprocal of the resistance of the material, and is expressed in mhos.

CONTINUOUS WAVES. Radio waves which maintain a constant amplitude and a constant frequency.

CONTROL GRID (G_1). The electrode of a vacuum tube other than a diode upon which the signal voltage is impressed in order to control the plate current.

CONTROL—GRID—PLATE TRANSCONDUCTANCE. See Transconductance.

CONVERSION TRANSCONDUCTANCE (g_c). A characteristic associated with the mixer function of vacuum tubes, and used in the same manner as transconductance is used. It is the ratio of the IF current in the primary of the first IF transformer to the RF signal voltage producing it.

CONVERTER. See Mixer.

CONVERTER TUBE. A multielement vacuum tube used both as a mixer and as an oscillator in a superheterodyne receiver. It creates a

local oscillator frequency and combines it with an incoming signal to produce an intermediate frequency.

COUNTING CIRCUIT. A circuit which receives uniform pulses representing units to be counted and produces a voltage in proportion to their frequency.

COUPLED IMPEDANCE. The effect produced in the primary winding of a transformer by the current flowing in the secondary winding.

COUPLING. The association of two circuits in such a way that energy may be transferred from one to the other.

COUPLING ELEMENT. The means by which energy is transferred from one circuit to another; the common impedance necessary for coupling.

CRITICAL COUPLING. The degree of coupling which provides the maximum transfer of energy between two resonant circuits at the resonant frequency.

CRYSTAL (Xtal). (1) A natural substance, such as quartz or tourmaline, which is capable of producing a voltage stress when under pressure, or producing pressure when under an applied voltage. Under stress it has the property of responding only to a given frequency when cut to a given thickness.

(2) A nonlinear element such as galena or silicon in which case the piezo-electric characteristic is not exhibited.

CRYSTAL MIXER. A device which employs the nonlinear characteristic of a crystal (non-piezo-electric type) and a point contact to mix two frequencies.

CRYSTAL OSCILLATOR. An oscillator circuit in which a piezoelectric crystal is used to control the frequency and to reduce frequency instability to a minimum.

CURRENT (I). Flow of electrons, measured in amperes.

CUT-OFF (co). The minimum value of negative grid bias which prevents the flow of plate current in a vacuum tube.

CUT-OFF LIMITING. Limiting the maximum output voltage of a vacuum-tube circuit by driving the grid beyond cut-off.

CYCLE. One complete positive and one complete negative alternation of a current or voltage waveshape.

DAMPED WAVES. Waves which decrease exponentially in amplitude.

DECOUPLING NETWORK. A network of capacitors, chokes, or resistors, placed in leads which are common to two or more circuits to prevent unwanted interstage coupling.

DEFLECTION SENSITIVITY (CRT). The quotient of the displacement of the electron beam at the place of impact by the change in the deflecting field. It is usually expressed in mil-

limeters per volt applied between the deflection electrodes, or in millimeters per gauss of the deflecting magnetic field.

DEGENERATION. The process whereby a part of the output signal of an amplifying device is returned to its input circuit in such a manner that it tends to cancel part of the input.

DE-IONIZATION POTENTIAL. The potential at which ionization of the gas within a gas-filled tube ceases and conduction stops.

DEMODULATION. See Detection.

DETECTION. The process of separating the modulation component from the received signal.

DIELECTRIC. An insulator; a term applied to the insulating material between the plates of a capacitor.

DIELECTRIC CONSTANT. The ratio of the capacitance of a capacitor with a dielectric between the electrodes to tne capacitance with air between the electrodes.

DIFFERENTIATING CIRCUIT. A circuit which produces an output voltage substantially in proportion to the rate of change of the input voltage.

DIODE. A two-electrode vacuum tube containing a cathode and a plate.

DIODE DETECTOR. A detector circuit employing a diode tube.

DIPOLE ANTENNA. Two metallic elements, each approximately one quarter wavelength long, which radiate RF energy fed to them by the transmission line.

DIRECTLY HEATED CATHODE. A filament cathode which carries its own heating current for electron emission, as distinguished from an indirectly heated cathode.

DIRECTOR (antenna). A parasitic antenna placed in front of a radiating element so that RF radiation is aided in the forward direction.

DISTORTION. The production of an output waveform which is not a true reproduction of the input waveform. Distortion may consist of irregularities in amplitude, frequency, or phase.

DISTRIBUTED CAPACITANCE. The capacitance that exists between the turns in a coil or choke, or between adjacent conductors or circuits, as distinguished from the capacitance which is concentrated in a capacitor.

DISTRIBUTED INDUCTANCE. The inductance that exists along the entire length of a conductor, as distinguished from the self-inductance which is concentrated in a coil.

DOORKNOB TUBE. A doorknob-shaped vacuum tube designed for ultra-high-frequency circuits. This tube has short electron transit time and low interelectrode capacitance, because of the close spacing and small size of electrodes.

DROPPING RESISTOR. A resistor used to decrease a given voltage to a lower value.

DRY ELECTROLYTIC CAPACITOR. An electrolytic capacitor using a paste instead of a liquid electrolyte. See Electrolytic Capacitor.

DYNAMIC CHARACTERISTICS. The relation between the instantaneous plate voltage and plate current of a vacuum tube as the voltage applied to the grid is moved; thus, the characteristics of a vacuum tube during operation.

DYNATRON. A negative resistance device; particularly, a tetrode operating on that portion of its i_p versus e_p characteristic where secondary emission exists to such an extent that an increase in plate voltage actually causes a decrease in plate current, and therefore, makes the circuit behave like a negative resistance.

ECCLES-JORDAN CIRCUIT (trigger circuit). A direct coupled multivibrator circuit possessing two conditions of stable equilibrium. Also known as a flip-flop circuit.

EFFECTIVE VALUE. The equivalent heating value of an alternating current or voltage, as compared to a direct current or voltage. It is 0.707 times the peak value of a sine wave. It is also called the RMS value.

EFFICIENCY. The ratio of output to input power, generally expressed as a percentage.

ELECTRIC FIELD. A space in which an electric charge will experience a force exerted upon it.

ELECTRODE. A terminal at which electricity passes from one medium into another.

ELECTROLYTE. A water solution of a substance which is capable of conducting electricity. An electrolyte may be in the form of either a liquid or a paste.

ELECTROLYTIC CAPACITOR. A capacitor employing a metallic plate and an electrolyte as the second plate separated by a dielectric which is produced by electrochemical action.

ELECTROMAGNETIC FIELD. A space field in which electric and magnetic vectors at right angles to each other, travel in a direction at right angles to both.

ELECTRON. The negatively charged particles of matter. The smallest particle of matter.

ELECTRON EMISSION. The liberation of electrons from a body into space under the influence of heat, light, impact, chemical disintegration, or potential difference.

ELECTRONIC SWITCH. A circuit which causes a start-and-stop action or a switching action by electronic means.

ELECTRONIC VOLTMETER. See Vacuum Tube Voltmeter.

ELECTROSTATIC FIELD. The field of influence between two charged bodies.

EQUIVALENT CIRCUIT. A diagrammatic arrangement of coils, resistors, and capacitors, representing the effects of a more complicated circuit in order to permit easier analysis.

FARAD (f). The unit of capacitance.

FEEDBACK. A transfer of energy from the output circuit of a device back to its input.

FIELD. The space containing electric or magnetic lines of force.

FIELD INTENSITY. Electrical or magnetic strength of a field.

FILAMENT. See Directly Heated Cathode.

FILTER. A combination of circuit elements designed to pass a definite range of frequencies, attenuating all others.

FIRING POTENTIAL. The controlled potential at which conduction through a gas-filled tube begins.

FIRST DETECTOR. See Mixer.

FIXED BIAS. A bias voltage of constant value, such as one obtained from a battery, power supply, or generator.

FIXED CAPACITOR. A capacitor which has no provision for varying its capacitance.

FIXED RESISTOR. A resistor which has no provision for varying its resistance.

FLUORESCENCE. The property of emitting light as the immediate result of electronic bombardment.

FLY-BACK. The portion of the time base during which the spot is returning to the starting point. This is usually not seen on the screen of the cathode-ray tube because of gating action or the rapidity with which it occurs.

FREE ELECTRONS. Electrons which are loosely held and consequently tend to move at random among the atoms of the material.

FREE OSCILLATIONS. Oscillatory currents which continue to flow in a tuned circuit after the impressed voltage has been removed. Their frequency is the resonant frequency of the tuned circuit.

FREQUENCY (f). The number of complete cycles per second existing in any form of wave motion; such as the number of cycles per second of an alternating current.

FREQUENCY DISTORTION. Distortion which occurs as a result of failure to amplify or attenuate equally all frequencies present in a complex wave.

FREQUENCY MODULATION. See Modulation.

FREQUENCY STABILITY. The ability of an oscillator to maintain its operation at a constant frequency.

FULL-WAVE RECTIFIER CIRCUIT. A circuit which utilizes both the positive and the negative alternations of an alternating current to produce a direct current.

GAIN (A). The ratio of the output power, voltage, or current to the input power, voltage, or current, respectively.

GAS TUBE. A tube filled with gas at low pres-

sure in order to obtain certain desirable characteristics.

GATING (cathode-ray tube). Applying a rectangular voltage to the grid or cathode of a cathode-ray tube to sensitize it during the sweep time only.

GRID CURRENT. Current which flows between the cathode and the grid whenever the grid becomes positive with respect to the cathode.

GRID DETECTION. Detection by rectification in the grid circuit of a detector.

GRID LEAK. A high resistance connected across the grid capacitor or between the grid and the cathode to provide a dc path from grid to cathode and to limit the accumulation of charge on the grid.

GRID LIMITING. Limiting the positive grid voltage (minimum output voltage) of vacuum-tube circuit by means of a large series grid resistor.

GROUND. A metallic connection with the earth to establish ground potential. Also, a common return to a point of zero RF potential, such as the chassis of a receiver or a transmitter.

HALF-WAVE RECTIFICATION. The process of rectifying an alternating current wherein only one-half of the input cycle is passed and the other half is blocked by the action of the rectifier, thus producing pulsating direct current.

HARD TUBE. A high vacuum electron tube.

HARMONIC. An integral multiple of a fundamental frequency. (The second harmonic is twice the frequency of the fundamental or first harmonic.)

HARMONIC DISTORTION. See Amplitude Distortion.

HEATER. The tube element used to indirectly heat a cathode.

HENRY (h). The basic unit of inductance.

HELMHOLTZ COIL. A variometer having horizontal and vertical balanced coil windings, used to vary the angle of phase difference between any two similar waveforms of the same frequency.

HETERODYNE. To beat or mix two signals of different frequencies.

HIGH-FREQUENCY RESISTANCE. The resistance presented to the flow of high-frequency current. See Skin Effect.

HORN RADIATOR. Any open-ended device for concentrating energy from a waveguide and directing this energy into space.

HYSTERESIS. A lagging of the magnetic flux in a magnetic material behind the magnetizing force which is producing it.

IMAGE FREQUENCY. An undesired signal capable of beating with the local oscillator signal of a superheterodyne receiver which produces a difference frequency within the bandwidth of the IF channel.

IMPEDANCE (Z). The total opposition offered to the flow of an alternating current. It may consist of any combination of resistance, inductive reactance, and capacitive reactance.

IMPEDANCE COIL. See Choke.

IMPEDANCE COUPLING. The use of a tuned circuit or an impedance coil as the common coupling element between two circuits.

IMPULSE. Any force acting over a comparatively short period of time, such as a momentary rise in voltage.

INDIRECTLY HEATED CATHODE. A cathode which is brought to the temperature necessary for electron emission by a separate heater element. Compare Directly Heated Cathode.

INDUCTANCE (L). The property of a circuit which tends to oppose a change in the existing current.

INDUCTION. The act or process of producing voltage by the relative motion of a magnetic field across a conductor.

INDUCTIVE REACTANCE (X_L). The opposition to the flow of alternating or pulsating current caused by the inductance of a circuit. It is measured in ohms.

INDUCTOR. A circuit element designed so that its inductance is its most important electrical property; a coil.

INFINITE. Extending indefinitely; having innumerable parts, capable of endless division within itself.

IN PHASE. Applied to the condition that exists when two waves of the same frequency pass through their maximum and minimum values of like polarity at the same instant.

INSTANTANEOUS VALUE. The magnitude at any particular instant when a value is continually varying with respect to time.

INTEGRATING CIRCUIT. A circuit which produces an output voltage substantially in proportion to the frequency and amplitude of the input voltage.

INTENSIFY. To increase the brilliance of an image on the screen of a cathode-ray tube.

INTENSITY MODULATION. The control of the brilliance of the trace on the screen of a cathode-ray tube in conformity with the signal.

INTERELECTRODE CAPACITANCE. The capacitance existing between the electrodes in a vacuum tube.

INTERMEDIATE FREQUENCY (IF). The fixed frequency to which RF carrier waves are converted in a superheterodyne receiver.

INVERSE PEAK VOLTAGE. The highest instantaneous negative potential which the plate can acquire with respect to the cathode without danger of injuring the tube.

ION. An elementary particle of matter or a small group of such particles having a net positive or negative charge.

IONIZATION. Process by which ions are produced in solids, liquids, or gases.

IONIZATION POTENTIAL. The lowest potential at which ionization takes place within a gas-filled tube.

IONOSPHERE. A region composed of highly ionized layers of atmosphere from 70 to 250 miles above the surface of the earth.

KILO (k). A prefix meaning 1,000.

KILOCYCLE (kc). One thousand cycles; conversationally used to indicate 1,000 cycles per second.

KLYSTRON. A tube in which oscillations are generated by the bunching of electrons (that is, velocity modulation). This tube utilizes the transit time between two given electrodes to deliver pulsating energy to a cavity resonator in order to sustain oscillations within the cavity.

LAG. The amount one wave is behind another in time; expressed in electrical degrees.

LEAD. The opposite of lag. Also, a wire or connection.

LEAKAGE. The electrical loss due to poor insulation.

LECHER LINE. A section of open-wire transmission line used for measurements of standing waves.

LIMITING. Removal by electronic means of one or both extremities of a waveform at a predetermined level.

LINEAR. Having an output which varies in direct proportion to the input.

LINE-BALANCE CONVERTER. A device used at the end of a coaxial line to isolate the outer conductor from ground.

LOAD. The impedance to which energy is being supplied.

LOCAL OSCILLATOR. The oscillator used in a superheterodyne receiver, the output of which is mixed with the desired RF carrier to form the intermediate frequency.

LOOSE COUPLING. Less than critical coupling; coupling providing little transfer of energy.

MAGNETIC CIRCUIT. The complete path of magnetic lines of force.

MAGNETIC FIELD (H). The space in which a magnetic force exists.

MAGNETRON. A vacuum-tube oscillator containing two electrodes, in which the flow of electrons from cathode to anode is controlled by an externally applied magnetic field.

MATCHED IMPEDANCE. The condition which exists when two coupled circuits are so adjusted that their impedances are equal.

MEG (mega) (M). A prefix meaning one million.

MEGACYCLE (Mc). One million cycles. Used conversationally to mean 1,000,000 cycles per second.

METALLIC INSULATOR. A shorted quarter-wave section of a transmission line which acts as an electrical insulator at a frequency corresponding to its quarter-wave length.

MHO. The unit of conductance.

MICRO (u). A prefix meaning one-millionth.

MICROSECOND (us). One-millionth of a second.

MILLI (m). A prefix meaning one-thousandth.

MILLIAMPERE (ma). One-thousandth of an ampere.

MIXER. A vacuum tube or crystal and suitable circuit used to combine the incoming and local-oscillator frequencies to produce an intermediate frequency. See Beat Frequency.

MODULATION. The process of impressing intelligence on an RF carrier in such a manner as to vary: the amplitude of the resultant wave (amplitude modulation), the frequency of the resultant wave (frequency modulation) or the phase of the resultant wave (phase modulation).

MODULATOR. The circuit which provides the signal, that varies the amplitude, frequency or phase of the resultant wave in a transmitter.

MULTIELECTRODE TUBE. A vacuum tube containing more than three electrodes associated with a single electron stream.

MULTIUNIT TUBE. A vacuum tube containing within one envelope two or more groups of electrodes, each associated with separate electron streams.

MULTIVIBRATOR. A type of relaxation oscillator for the generation of nonsinusoidal waves in which the output of each of its two tubes is coupled to the input of the other to sustain oscillations.

MUTUAL CONDUCTANCE (g_m). See Transconductance.

MUTUAL INDUCTANCE. A circuit property existing when the relative position of two inductors causes the magnetic lines of force from one to link with the turns of the other.

NEGATIVE FEEDBACK. See Degeneration.

NEON BULB. A glass bulb containing two electrodes in neon gas at low pressure.

NETWORK. Any electrical circuit containing two or more interconnected elements.

NEUTRALIZATION. The process of nullifying the voltage fed back through the interelectrode capacitance of an amplifier tube, by providing an equal voltage of opposite phase; generally necessary only with triode tubes.

NODE. A zero point; specifically, a current node is a point of zero current and a voltage node is a point of zero voltage.

NONINDUCTIVE CAPACITOR. A capacitor in which the inductive effects at high frequencies are reduced to the minimum.

NONINDUCTIVE CIRCUIT. A circuit in which inductance is reduced to a minimum or negligible value.

NONLINEAR. Having an output which does not

vary in direct proportion to the input.

OHM (Ω). The unit of electrical resistance.

OPEN CIRCUIT. A circuit which does not provide a complete path for the flow of current.

OPTIMUM COUPLING. See Critical Coupling.

OSCILLATOR. A circuit capable of converting direct current into alternating current of a frequency determined by the constants of the circuit. It generally uses a vacuum tube.

OSCILLATORY CIRCUIT. A circuit in which oscillations can be generated or sustained.

OSCILLOGRAPH. See Oscilloscope.

OSCILLOSCOPE. An instrument for showing, visually, graphical representations of the waveforms encountered in electrical circuits.

OVERDRIVEN AMPLIFIER. An amplifier designed to distort the input signal waveform by a combination of cut-off limiting and saturation limiting.

OVERLOAD. A load greater than the rated load of an electrical device.

PARALLEL FEED. Application of a dc voltage to the plate or grid of a tube in parallel with an ac circuit so that the dc and ac components flow in separate paths. Also called shunt feed.

PARALLEL-RESONANT CIRCUIT. A resonant circuit in which the applied voltage is connected across a parallel circuit formed by a capacitor and an inductor.

PARAPHASE AMPLIFIER. An amplifier which converts a single input into a push-pull output.

PARASITIC SUPPRESSOR. A resistor in a vacuum-tube circuit to prevent unwanted oscillations.

PEAKING CIRCUIT. A type of circuit which converts an input to a peaked output waveform.

PEAK PLATE CURRENT. The maximum instantaneous plate current passing through a tube.

PEAK VALUE. The maximum instantaneous value of a varying current, voltage, or power. It is equal to 1.414 times the effective value of a sine wave.

PENTODE. A five-electrode vacuum tube containing a cathode, control grid, screen grid, suppressor grid, and plate.

PHASE DIFFERENCE. The time in electrical degrees by which one wave leads or lags another.

PHASE INVERSION. A phase difference of 180° between two similar waveshapes of the same frequency.

PHASE-SPLITTING CIRCUIT. A circuit which produces from the same input waveform two output waveforms which differ in phase from each other.

PHOSPHORESCENCE. The property of emitting light for some time after excitation by electronic bombardment.

PIEZOELECTRIC EFFECT. The effect of producing a voltage by placing a stress, either by compression, by expansion, or by twisting, on a crystal, and, conversely, the effect of producing a stress in a crystal by applying a voltage to it.

PLATE (P). The principal electrode in a tube to which the electron stream is attracted. See Anode.

PLATE CIRCUIT. The complete electrical circuit connecting the cathode and plate of a vacuum tube.

PLATE CURRENT. The current flowing in the plate circuit of a vacuum tube.

PLATE DETECTION. The operation of a vacuum tube detector at or near cut-off so that the input signal is rectified in the plate circuit.

PLATE DISSIPATION. The power in watts consumed at the plate in the form of heat.

PLATE EFFICIENCY. The ratio of the ac power output from a tube to the average dc power supplied to the plate circuit.

PLATE IMPEDANCE. See Plate Resistance.

PLATE-LOAD IMPEDANCE (R_L or Z_L). The impedance in the plate circuit across which the output signal voltage is developed by the alternating component of the plate current.

PLATE MODULATION. Amplitude modulation of a class C RF amplifier by varying the plate voltage in accordance with the modulating signal.

PLATE RESISTANCE (r_p). The internal resistance to the flow of alternating current between the cathode and plate of tube. It is equal to a small change in plate voltage divided by the corresponding change in plate current, and is expressed in ohms. It is also called ac resistance, internal impedance, plate impedance, and dynamic plate impedance. The static plate resistance, or resistance to the flow of direct current is a different value. It is denoted by R_b.

POSITIVE FEEDBACK. See Regeneration.

POTENTIOMETER. A variable divider; a resistor which has a variable contact arm so that any portion of the potential applied between its ends may be selected.

POWER. The rate of doing work or the rate of expending energy. The unit of electrical power is the watt.

POWER AMPLIFICATION. The process of amplifying a signal to produce a gain in power, as distinguished from voltage amplification. The gain in the ratio of the alternating power output to the alternating power input of an amplifier.

POWER FACTOR. The ratio of the actual power of an alternating or pulsating current, as measured by a wattmeter, to the apparent power, as indicated by ammeter and voltmeter read-

ings. The power factor if an inductor, capacitor, or insulator is an expression of the losses.

POWER TUBE. A vacuum tube designed to handle a greater amount of power than the ordinary voltage-amplifying tube.

PRIMARY CIRCUIT. The first, in electrical order, of two or more coupled circuits, in which a change in current induces a voltage in the other or secondary circuits; such as the primary winding of a transformer.

PROPAGATION. See Wave Propagation.

PULSATING CURRENT. A unidirectional current which increases and decreases in magnitude.

PUSH-PULL CIRCUIT. A push-pull circuit usually refers to an amplifier circuit using two vacuum tubes in such a fashion that when one vacuum tube is operating on a positive alternation, the other vacuum tube operates on a negative alternation.

Q. The symbol used to denote a quantity of electrical charge.

Q. The figure of merit of efficiency of a circuit or coil. Numerically it is equal to the inductive reactance divided by the resistance of the circuit or coil.

RADIATE. To send out energy, such as RF waves, into space.

RADIATION RESISTANCE. A fictitious resistance which would dissipate the same power that the antenna dissipates.

RADIO FREQUENCY (RF). Any frequency of electrical energy capable of propagation into space. Radio frequencies normally are much higher than sound wave frequencies.

RADIO-FREQUENCY AMPLIFICATION. The amplification of a radio wave by a receiver before detection, or by a transmitter before radiation.

RADIO-FREQUENCY CHOKE (RFC). An air-core or powdered iron core coil used to impede the flow of RF currents.

RADIO-FREQUENCY COMPONENT. See Carrier.

RATIO. The value obtained by dividing one number by another, indicating their relative proportions.

REACTANCE (X). The opposition offered to the flow of an alternating current by the inductance, capacitance, or both, in any circuit.

RECIPROCAL. The value obtained by dividing the number 1 by any quantity.

RECTIFIER. A device used to change alternating current to unidirectional current.

REFLECTED IMPEDANCE. See Coupled Impedance.

REFLECTION. The turning back of a radio wave caused by re-radiation from any conducting surface which is large in comparison to the wavelength of the radio wave.

REFLECTOR. A metallic object placed behind a radiating antenna to prevent RF radiation in an undesired direction and to reinforce radiation in a desired direction.

REGENERATION. The process of returning a part of the output signal of an amplifier to its input circuit in such a manner that it reinforces the grid excitation and thereby increases the total amplification.

REGULATION (voltage). The ratio of the change in voltage due to a load to the open-circuit voltage, expressed in per cent.

RELAXATION OSCILLATOR. A circuit for the generation of nonsinusoidal waves by gradually storing and quickly releasing energy either in the electric field of a capacitor or in the magnetic field of an inductor.

RELUCTANCE. The opposition to magnetic flux.

RESISTANCE (R). The opposition to the flow of current caused by the nature and physical dimensions of a conductor.

RESISTOR. A circuit element whose chief characteristic is resistance; used to oppose the flow of current.

RESONANCE. The condition existing in a circuit in which the inductive and capacitive reactances cancel.

RESONANCE CURVE. A graphical representation of the manner in which a resonant circuit responds to various frequencies at and near the resonant frequency.

RHEOSTAT. A variable resistor.

RIPPLE VOLTAGE. The fluctuations in the output voltage of a rectifier, filter, or generator.

RMS. Abbreviation of root mean square. See Effective Value.

SATURATION. The condition existing in any circuit when an increase in the driving signal produces no further change in the resultant effect.

SATURATION LIMITING. Limiting the minimum output voltage of a vacuum-tube circuit by operating the tube in the region of plate-current saturation (not to be confused with emission saturation).

SATURATION POINT. The point beyond which an increase in either grid voltage, plate voltage, or both, produces no increase in the existing plate current.

SCREEN DISSIPATION. The power dissipated in the form of heat on the screen grid as the result of bombardment by the electron stream.

SCREEN GRID (G2). An electrode placed between the control grid and the plate of a vacuum tube to reduce interelectrode capacitance.

SECONDARY. The output coil of a transformer. See Primary Circuit.

SECONDARY EMISSION. The emission of elec-

trons knocked loose from the plate, grid, or fluorescent screen of a vacuum tube by the impact or bombardment of electrons arriving from the cathode.

SELECTIVITY. The degree to which a receiver is capable of discriminating between signals of different carrier frequencies.

SELF-BIAS. The bias of a tube created by the voltage drop developed across a resistor through which its cathode current flows.

SELF-EXCITED OSCILLATOR. An oscillator depending on its resonant circuits for frequency determination. See Crystal Oscillator.

SELF-INDUCTION. The production of a counterelectromotive force in a conductor when its own magnetic field collapses or expands with a change in current in the conductor.

SENSITIVITY. The degree of response of a circuit to signals of the frequency to which it is tuned.

SERIES FEED. Application of the dc voltage to the plate or grid of a tube through the same impedance in which the alternating current flows. Compare Parallel Feed.

SERIES RESONANCE. The condition existing in a circuit when the source of voltage is in series with an inductor and capacitor whose reactances cancel each other at the applied frequency and thus reduce the impedance to minimum.

SERIES-RESONANT CIRCUIT. A resonant circuit in which the capacitor and the inductor are in series with the applied voltage.

SHIELDING. A metallic covering used to prevent magnetic or electrostatic coupling between adjacent circuits.

SHORT-CIRCUIT. A low-impedance or zero-impedance path between two points.

SHUNT. Parallel. A parallel resistor placed in an ammeter to increase its range.

SHUNT FEED. See Parallel Feed.

SINE WAVE. The curve traced by the projection on a uniform time scale of the end of a rotating arm, or vector. Also known as a sinusoidal wave.

SKIN EFFECT. The tendency of alternating currents to flow near the surface of a conductor thus being restricted to a small part of the total cross-sectional area. This effect increases the resistance and becomes more marked as the frequency rises.

SOFT TUBE. A vacuum tube, the characteristics of which are adversely affected by the presence of gas in the tube; not to be confused with tubes designed to operate with gas inside them.

SOLENOID. A multiturn coil of wire wound in a uniform layer or layers on a hollow cylindrical form.

SPACE CHARGE. The cloud of electrons existing in the space between the cathode and plate in a vacuum tube, formed by the electrons emitted from the cathode in excess of those immediately attracted to the plate.

SPACE CURRENT. The total current flowing between the cathode and all the other electrodes in a tube. This includes the plate current, grid current, screen-grid current, and any other electrode current which may be present.

STABILITY. Freedom from undesired variation.

STANDING WAVE. A distribution of current and voltage on a transmission line formed by two sets of waves traveling in opposite directions and characterized by the presence of a number of points of successive maxima and minima in the distribution curves.

STATIC. A fixed nonvarying condition; without motion.

STATIC CHARACTERISTICS. The characteristics of a tube with no output load and with dc potentials applied to the grid and plate.

SUPERHETERODYNE. A receiver in which the incoming signal is mixed with a locally generated signal to produce a predetermined intermediate frequency.

SUPPRESSOR GRID (G3). An electrode used in a vacuum tube to minimize the harmful effects of secondary emission from the plate.

SURGE. Sudden changes of current or voltage in a circuit.

SURGE IMPEDANCE. See Characteristic Impedance.

SWEEP CIRCUIT. The part of a cathode-ray oscilloscope which provides a time-reference base.

SWING. The variation in frequency or amplitude of an electrical quantity.

SWINGING CHOKE. A choke with an effective inductance which varies with the amount of current passing through it. It is used in some power-supply filter circuits.

SYNCHRONOUS. Happening at the same time; having the same period and phase.

TANK CIRCUIT. See Parallel-resonant Circuit.

TETRODE. A four-electrode vacuum tube containing a cathode, control grid, screen grid, and plate.

THERMIONIC EMISSION. Electron emission caused by heating an emitter.

THERMOCOUPLE AMMETER. An ammeter which operates by means of a voltage produced by the heating effect of a current passed through the junction of two dissimilar metals. It is used for RF measurements.

THYRATRON. A hot-cathode, gas-discharge tube in which one or more electrodes are used to control electrostatically the starting of an unidirectional flow of current.

TIGHT COUPLING. Degree of coupling in which practically all of the magnetic lines of force produced by one coil link a second coil.

TRACE. A visible line or lines appearing on the screen of a cathode-ray tube in operation.

TRANSCONDUCTANCE (g_m). The ratio of the change in plate current to the change in grid voltage producing this change in plate current, while all other electrode voltages remain constant.

TRANSFORMER. A device composed of two or more coils, linked by magnetic lines of force, used to transfer energy from one circuit to another.

TRANSIENT. The voltage or current which exists as the result of a change from one steady-state condition to another.

TRANSIT TIME. The time which electrons take to travel between the cathode and the plate of a vacuum tube.

TRANSMISSION LINES. Any conductor or system of conductors used to carry electrical energy from its source to a load.

TRIGGERING. Starting an action in another circuit, which then functions for a time under its own control.

TRIODE. A three-electrode vacuum tube, containing a cathode, control grid, and plate.

TUNED CIRCUIT. A resonant circuit.

TUNING. The process of adjusting a radio circuit so that it resonates at the desired frequency.

UNBALANCED LINE. A transmission line in which the voltages on the two conductors are not equal with respect to ground; for example, a coaxial line.

UNIDIRECTIONAL. In one direction only.

VACUUM-TUBE VOLTMETER (VTVM). A device which uses either the amplifier characteristic or the rectifier characteristic of a vacuum tube or both to measure either dc or ac voltages. Its input impedance is very high, and the current used to actuate the meter movement is not taken from the circuit being measured. It can be used to obtain accurate measurements in sensitive circuits.

VARIABLE-U TUBE. A vacuum tube in which the control grid is irregularly spaced, so that the grid exercises a different amount of control on the electron stream at different points within its operating range.

VARIOCOUPLER. Two independent inductors, so arranged mechanically that their mutual inductance (coupling) can be varied.

VARIOMETER. A variocoupler having its two coils connected in series, and so mounted that the movable coil may be rotated within the fixed coil, thus changing the total inductance of the unit.

VECTOR. A line used to represent both direction and magnitude.

VELOCITY MODULATION. A method of modulation in which the input signal voltage is used to change the velocity of electrons in a constant current electron beam so that the electrons are grouped into bunches.

VIDEO AMPLIFIER. A circuit capable of amplifying a very wide range of frequencies, including and exceeding the audio band of frequencies.

VOLT (V). The unit of electrical potential.

VOLTAGE AMPLIFICATION. The process of amplifying a signal to produce a gain in voltage. The voltage gain of an amplifier is the ratio of its alternating-voltage output to its alternating-voltage input.

VOLTAGE DIVIDER. An impedance connected across a voltage source. The load is connected across a fraction of this impedance so that the load voltage is substantially in proportion to this fraction.

VOLTAGE DOUBLER. A method of increasing the voltage by rectifying both halves of a cycle and causing the outputs of both halves to be additive.

VOLTAGE REGULATION. A measure of the degree to which a power source maintains its output-voltage stability under varying load conditions.

WATT (w). The unit of electrical power.

WAVE. Loosely, an electromagnetic impulse, periodically changing in intensity and traveling through space. More specifically, the graphical representation of the intensity of that impulse over a period of time.

WAVEFORM. The shape of the wave obtained when instantaneous values of an ac quantity are plotted against time in rectangular coordinates.

WAVELENGTH (λ). The distance, usually expressed in meters, traveled by a wave during the time interval of one complete cycle. It is equal to the velocity divided by the frequency.

WAVE PROPAGATION. The transmission of RF energy through space.

WIEN-BRIDGE CIRCUIT. A circuit in which the various values of capacitance and resistance are made to balance with each other at a certain frequency.

X. The symbol for reactance.

Z. The symbol for impedance.